04001016
G. Yashwanth.

40. 3 Stage Rocket

	stage 1	stage 2	stage 3
m_s	6,98,390	69,840	21,950
m_p	47,61,750	698,400	69,480
$\dot{m}$	55,300	5,533	700
Specific Impulse	230	286	286

Payload = 35400 kg.

Part (a): Find Burnout velocity

$V_{e_1} = 230g$

$V_{e_2} = 286g$

$V_{e_3} = 286g$.

$$V_f = -\sum_{i=1}^{3} V_{ei} \ln\left(\varepsilon_i + (1-\varepsilon_i)\pi_i\right)$$

$$\varepsilon_1 = \frac{6,98,390}{6,98,390 + 47,61,750} = 0.1279$$

$$\varepsilon_2 = \frac{69840}{69840 + 698400} = 0.0909$$

$$\varepsilon_3 = \frac{21950}{69480 + 21950} = 0.2401$$

$$\pi_3 = \frac{35400}{35400 + 21950 + 69480} = \frac{35400}{126830} = 0.2791$$

$$\pi_2 = \frac{126830}{126830 + 69840 + 698400} = 0.1417$$

$$\pi_1 = \frac{895070}{895070 + 698390 + 47,61,750} = 0.1408$$

$\therefore V_f = 230g \times 1.38353 + 286g \times 1.1514 +$
$286g \times 0.7936$

$= 9590$ m/sec

Burnout Velocity = 9590 m/sec

(b) Structural mass increased by 7% in stage 2 & 3
Specific Impulse is 295
Assuming that mass of propellant Remain same.

Let new payload ie $m^* = P$

	Stage 1	Stage 2	Stage 3
m_s	698390	74728	23486
m_p	4761750	698400	69480

$$\Pi_3 = \frac{P}{P + 92966}$$

$$\Pi_2 = \frac{P + 92966}{P + 866094}$$

$$\Pi_1 = \frac{P + 866094}{P + 6326234}$$

$\varepsilon_1 = 0.1279$
$\varepsilon_2 = 0.0966$
$\varepsilon_3 = 0.2526$

$$9590 = -\left[230g \times \ln\left(\varepsilon_1 + (1-\varepsilon_1)\Pi_1\right) + 295g \ln\left(\varepsilon_2 + (1-\varepsilon_2)\Pi_2\right) + 295g \ln\left(\varepsilon_3 + (1-\varepsilon_3)\Pi_3\right)\right]$$

Substituting $\varepsilon_1\ \varepsilon_2\ \varepsilon_3,\ \Pi_1\ \Pi_2\ \Pi_3$ from above and solving, we get $P = 37611$ kg

how did you solve this?

$\therefore$ Increase in Payload = $37611 - 35400 = 2211$ kg

$= 2211$ kg

YOU HAVE TO USE TRADE-OFF RATIOS FOR THIS PROBLEM

(c) Velocity Increment for last two stages:

$$V_{f_3} - V_{f_1} = (V_{f_3} - V_{f_2}) + (V_{f_2} - V_{f_1})$$
$$= 9590 - 3118 = \underline{6472 \text{ m/sec}}.$$

Re design - third stage

Let structural mass be m_{s_3}

Let propellant mass be m_{p_3}.

overall payload ratio for last two stage $= \Pi_2 \Pi_3$

$$\Pi_2 \Pi_3 = \frac{m^*}{(m_{s_2} + m_{p_2}) + m_{s_3} + m_{p_3} + m^*} \rightarrow \text{maximize}$$

given: $\dfrac{m_{s_2}}{m_{p_2} + m_{s_2}} = \dfrac{m_{s_3}}{m_{s_3} + m_{p_3}} = \dfrac{1}{11}$

$\Rightarrow \quad m_{s_3} + m_{p_3} = 11 m_{s_3} \quad \Rightarrow \quad m_{p_3} = 10 m_{s_3}$

$$\Rightarrow \text{maximize} \quad \frac{m^*}{768240 + 11 m_{s_3} + m^*}$$

subject to:

$$V_{f_3} - V_{f_1} = -V_{e_3} \ln(\varepsilon_3 + (1-\varepsilon_3)\Pi_3) - V_{e_2} \ln(\varepsilon_2 + (1-\varepsilon_2)\Pi_2)$$
$$6472 = -2869 \ln$$

You simply have to use the solution for the ~~optimall~~ optimally staged rocket that we derived in class.

$$\varepsilon_3 = \frac{m_{s_3}}{m_{s_3} + m_{p_3}} = \frac{1}{11} \qquad \Pi_3 = \frac{m^*}{11 m_{s_3} + m^*}$$

$$\varepsilon_2 = \frac{1}{11} \qquad \Pi_2 = \frac{11 m_{s_3} + m^*}{(11 m_{s_3} + m^*) + 768240}$$

$$6472 = -2869 \ln \left[0.0909 + 0.9091 \times \frac{(11 m_{s_3} + m^{+})}{(11 m_{s_3} + m^{+} + 768240)} \right]$$

$$- 2869 \ln \left[0.0909 + 0.9091 \times \frac{m^{+}}{115 m_{s_3} + m^{+}} \right]$$

Q. 12/3/08. Soln. to tutorial problem.

MAYUR SINGH
04D01003

But the problem has a parallel burning phase,

Q:37 → In ideal condition, there is no overlapping of stages.

Let us assume vehicle ascent at constant thrust during a stage and under uniform gravity. → for maximum possible burnout speed, assume gravity = 0.

for maximum burnout speed of PSLV, thrust should be maintained constant = max. thrust of each stage.

The final burnout ~~vs~~speed after 4th stage

$$V_b = V_{b_3} + \left[-g t_{b_4} + V_{e_4} \ln\left(\frac{m_{i_4}}{m_{f_4}}\right)\right]$$

$m_{f_4} = m_* + m_{s_4}$.

$m_{f_4} = m_*$ = for payload weight

Similarly,

$$V_{b_3} = V_{b_2} + \left[-g t_{b_3} + V_{e_3} \ln\left(\frac{m_{i_3}}{m_{f_3}}\right)\right]$$

$$V_{b_2} = V_{b_1} + \left[-g t_{b_2} + V_{e_2} \ln\left(\frac{m_{i_2}}{m_{f_2}}\right)\right]$$

$$V_{b_1} = V_1 + \left[-g (t_{b_1} - t_{b_b}) + V_{e_1} \ln\left(\frac{m_{i_1}}{m_{f_1}}\right)\right]$$

$$V_1 = 0 - g t_{b_b} + \bar{V}_e \ln\left(\frac{m_i}{m_f}\right)$$

~~gl~~ where t_{b_b} → burnout time for booster

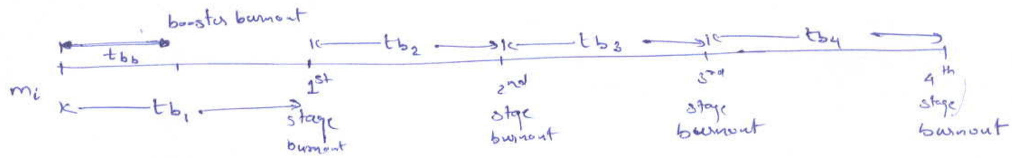
& $\bar{V}_e$ = equivalent relative exit velocity

$$= \cancel{6 V_{e_b} + V} \; \frac{6 \cdot \dot{m}_b \cdot V_{e_b} + \dot{m}_1 \cdot V_{e_1}}{\dot{m}_b + \dot{m}_1}$$

~~$m_f = m_{i_1}$, $m_{f_1} = m_{i_2}$~~

$m_* = (1050 + 107 + 45)\ kg = 1202\ kg$

$m_i = m_* + m_{p_1} + m_{s_1} + m_{p_b} + m_{s_b} + m_{p_2} + m_{s_2} + m_{s_3} + m_{p_3} + m_{p_4} + m_{s_4}$



$m_i = (1.202 + 229 + 46 + 8.4 + 2.89)$ ton $\Big\}$ → For which stage?

$m_i = 287.492$ ton

m_f = mass when after t_{bb}

$= m_i - \cancel{13 m_{p_1}}\ \&\ \dot{m}_{p_1} \times (t_{b_1} + t_{bb})$

$= 287.492 - \frac{138}{107.4} \cdot (\cancel{107.4} + 45) - 54$

$m_f = \cancel{207.313\ ton}\ \cancel{229.67\ ton}\ 175.67$ ton

$V_{e_b} = \frac{T_b}{\dot{m}_b} = \frac{662 \times 45}{9} = 3310$ m/sec

$V_{e_1} = \frac{T_1}{\dot{m}_{p_1}} = \frac{4628 \times 107.4}{138} = 3601.79$ m/sec

$\bar{V}_e = \frac{6 \times T_b + T_1}{\dot{m}_{p_1} + \dot{m}_b} = \frac{6 \times 662 + 4628}{\frac{9}{45} + \frac{138}{107.4}} = 5791.57$ m/sec

$\Rightarrow V_1 = -9.8 \times 45 + 5791.57\ \cancel{\ln\left(\frac{287.492}{229.698}\right)}\ \ln\left(\frac{287.492}{175.67}\right)$

~~$V_1 = 1452.64$ m/sec~~

~~$V_1 = 859.5$ m/sec~~ $V_1 = 2411.86$ m/sec

$m_{i_1} = m_i - m_{p_b} - \dot{m}_{p_1} \times t_b = m_f$

$= 175.67$ ton

$m_{f_1} = m_i - m_{p_b} - m_{i_1}$

$= 287.492 - 54 - 138$

$= 95.492$ ton

$V_{b_1} \Rightarrow$ Burnout ~~velocity~~ speed ~~after~~ of 1st stage

$$V_{b_1} = V_1 + \left[-g\,(t_{b_1} - t_{b_b}) + V_{e_1} \ln\left(\frac{m_{i_1}}{m_{f_1}}\right) \right]$$

$$= 2411.86 + \left[9.8\,(107.4 - 45) + 3601.79 \ln\left(\frac{175.67}{95.492}\right) \right]$$

$V_{b_1} = 3995.86$ m/sec

~~Burnout velocity after~~

Burnout speed of 2nd stage

$$V_{b_2} = V_{b_1} - g t_{b_2} + V_{e_2} \ln\left(\frac{m_{i_2}}{m_{f_2}}\right)$$

$t_{b_2} = 163$ sec

$V_{e_2} = \frac{725}{40.6} \times 163$

$= 2910.71$ m/sec

$m_{i_2} = m_i - (m_{p_1} + m_{p_b} + m_{s_1} + m_{s_b}) = m_{f_1} - m_{s_1} - m_{s_b}$

$= 287.492 - 229$

$= 58.492$ ton

$m_{f_2} = m_{i_2} - m_{p_2}$

$= 58.492 - 40.6$

$= 17.892$ ton

$\Rightarrow v_{b_2} = 3995.86 - 9.8 \times 163 + 2910.71 \ln\left(\frac{58.492}{17.892}\right)$

$v_{b_2} = 5846.30$ m/sec

$v_{b_3} = v_{b_2} - g t_{b_3} + \bar{v}_{e_3} \ln\left(\frac{m_{i_3}}{m_{f_3}}\right)$

$t_{b_3} = 76$ sec

$v_{e_3} = \frac{340}{7.2} \times 76$

$= 3588.89$ m/sec

$m_{i_3} = m_{f_2} - m_{s_2}$

$= 58.492 - 46$

$= 12.492$ ton

$m_{f_3} = m_{i_3} - m_{p_3}$

$= 12.492 - 7.2$

$= 5.292$ ton

$v_{b_3} = \cancel{3588.89}\ 5846.30 - 9.8 \times 76 + 3588.89 \ln\left(\frac{12.492}{5.292}\right)$

$= 8183.96$ m/sec

$v_b = v_{b_4} = v_{b_3} - g t_{b_4} + \bar{v}_{e_4} \ln\left(\frac{m_{i_4}}{m_{f_4}}\right)$

$t_{b_4} = 415$ sec

there are 2 engines in 4^{th} stage

i.e. parallel combustion

1 ton ~~each~~ in each

$\dot{m} = \frac{14.8 \times 415}{2}$

$\bar{v}_{e_4} = \frac{2 \times \dot{m} v_{e_4}}{} \quad \frac{2 \times v_{e_4} \cdot \dot{m}}{\dot{m} + \dot{m}} = v_{e_4}$

$= 2 \times \left\{\frac{14.8}{2} \times 415\right\} = 3071$ m/sec

$m_{i_4} = m_{f_3} - m_{s_3}$

$= 12.492 - 8.4$

$= 4.092$

$m_{f_4} = m_{i_4} - m_{p_4}$

$= 4.092 - 2$

$= 2.092$

$\Rightarrow V_{b_4} = V_b = 8183.96 - 9.8 \times 415 + 3071 \ln\left(\frac{4.092}{2.092}\right)$

$= 6177.33$ m/sec

As it after observing burnout speed of each stage it is clear that maximum burnout speed in of 3rd stage

i.e, $(V_b)_{max} = V_{b_3} = 8183.96$ m/sec

This is not what is meant.
Maximum possible injection speed of the payload is asked.

(56) Tushar J. Malgunkar

* First stage – GS1 :

$$m_{i1} = m_{s1} + m_{p1} + m_{i2}$$
$$m_{f1} = m_{s1} + m_{i2}$$

m_{s1} = total structural mass of solid stage (S125) plus that of four L40 boosters strapped on to S125

m_{p1} = total propellant mass of S125 and 4 L40 boosters

$$\therefore \quad m_{s1} = (156-129) + 4(46-40) = 51 \text{ tonnes} = 51\times10^3 \text{ kg}$$
$$m_{p1} = 129 + (4\times40) = 289 \text{ tonnes} = 289\times10^3 \text{ kg}$$

not correct. S125 burns for 100 sec. L40s burn for 160 sec. Both are ejected together. You have to treat 60 sec. seperately.!

* Second stage – GS2 :

$$m_{i2} = m_{s2} + m_{p2} + m_{i3}$$
$$m_{f2} = m_{s2} + m_{i3}$$

Total mass = 42.8 tonnes propellant mass = 38 tonnes

$$\therefore \quad m_{p2} = 38 \text{ tonnes} = 38\times10^3 \text{ kg}$$
$$m_{s2} = (42.8 - 38) = 4.8 \text{ tonnes} = 4.8\times10^3 \text{ kg}$$

2nd stage burns 38 tonnes of propellant in 150 seconds

* Third stage – GS3 :

$$m_{i3} = m_{s3} + m_{p3} + m^*$$
$$m_{f3} = m_{s3} + m^*$$

Total mass = 15 tonnes propellant mass = 12.5 tonnes

$$\therefore \quad m_{p3} = 12.5 \text{ tonnes} = 12.5\times10^3 \text{ kg}$$
$$m_{s3} = (15 - 12.5) = 2.5 \text{ tonnes} = 2.5\times10^3 \text{ kg}$$
$$m^* = 1530 \text{ kg}.$$

$\therefore \quad m_{i3} = (2500 + 12500 + 1530) = 16530 \text{ kg}$

$m_{f3} = 2500 + 1550 = 4030 \text{ kg}$

$\therefore \quad m_{i2} = (38000 + 4800 + 16530) = 59330 \text{ kg}$

$m_{f2} = 21330 \text{ kg}.$

$m_{i1} = 399{,}330 \text{ kg}.$

$m_{f1} = 110{,}330 \text{ kg}$

* To find relative exit velocity at each stage

1st stage $\rightarrow \quad F_{net} = F_{125} + 4\,F_{140}$

$$\dot{m}_{tot}\,\bar{V}_{e1} = 4700 + (4 \times 680) = 7420 \text{ KN}$$

$$\dot{m}_{tot.} = \left(\frac{129}{102} \times 10^3 + 4 \times \frac{40 \times 10^3}{160}\right)$$

$$= 10^3\,(1.29 + 1) = 2.29 \times 10^3 \text{ kg/sec}$$

$$2.29 \times \cancel{10^3}\,\bar{V}_{e1} = 7420 \times \cancel{10^3} \text{ N}$$

$$\boxed{\bar{V}_{e1} = 3240.17 \text{ m/sec.}}$$

2nd stage $\rightarrow \quad V_{e2} = \frac{F_2}{\dot{m}_2} = \frac{720 \times 10^3}{\left(\frac{38 \times 10^3}{150}\right)}$

$$\boxed{V_{e2} = 2842.10 \text{ m/sec}}$$

3rd stage → $V_{e3} = \frac{F_3}{\dot{m}_3} = \frac{73.5 \times 10^3}{\left(\frac{12.5 \times 10^3}{720}\right)}$

$$\boxed{V_{e3} = 4233.6 \text{ m/sec}}$$

Now $V_f = \sum_{k=1}^{3} V_{ek} \ln\left(\frac{m_{ik}}{m_{fk}}\right)$

$$= 8240.17 \ln\left(\frac{399830}{110,330}\right) + 2842.10 \ln\left(\frac{59330}{21830}\right) + 4233.6 \ln\left(\frac{16530}{4030}\right)$$

$$V_f = 4167.87 + 2907.46 + 5975.34$$

$$\boxed{V_f = 13050.67 \text{ m/sec}}$$

T. Sateesh
03001017

Problem # 42

$$V_f = \sum v_{ej} \ln\left[\frac{m_{ij}}{m_{fj}}\right]$$

$$m_{ij} = m_* + m_{pj} + m_{sj} + \sum_{i=j+1}^{N}(m_{si} + m_{pi})$$
$$m_{fj} = m_* + 0 + m_{sj} + \sum_{i=j+1}(m_{si} + m_{pi})$$

$$\frac{\partial m^*}{\partial v_{ej}} = \frac{-\partial V_f / \partial v_e}{\partial V_f / \partial m^*}$$ (partial derivatives rules)

$$\frac{\partial V_f}{\partial v_{ej}} = \sum_{??} \ln\left[\frac{m_{ij}}{m_{fj}}\right]$$ {(the summation of all those where exit vel. is changed)}

$$\frac{\partial V_f}{\partial m^*} = \sum_{j=1}^{N} v_{ej}\left[\frac{1}{m_{ij}} - \frac{1}{m_{fj}}\right]$$

$$\boxed{\Rightarrow \frac{\partial m^*}{\partial v_e} = -\sum \ln\left[\frac{m_{ij}}{m_{fj}}\right] \Big/ \sum_{j=1}^{N} v_{ej}\left[\frac{1}{m_{ij}} - \frac{1}{m_{fj}}\right]}$$

$m_{p_1} = 1167$ kg, $m_{s_1} = 113$ kg
$m_{p_2} = 415$ kg, $m_{s_2} = 41$ kg
$m^* = 150$ kg
$I_{sp} = 282$ sec ; $\Delta I_{sp} = 10$ s.

$g \cdot I_{sp} = V_e$ $\Rightarrow V_e = 282 \times 9.8 = 2763.6$ m/s

$m_{1i} = 1167 + 113 + 415 + 41 + 150 = 1886$ kg
$m_{1f} = \cancel{1167} + 113 + 415 + 41 + 150 = 719$ kg

$m_{2i} = 415 + 41 + 150 = 606$ kg
$m_{2f} = 150 + 41 = 191$ kg

Suppose I_{sp} for 1st stage is ~~changed~~ improved by 10 s

$$\frac{\partial m^*}{\partial v_{e_1}} = \frac{-\ln\left[\frac{1886}{719}\right]}{2763.6\left\{\left(\frac{1}{1886} - \frac{1}{719}\right) + \left(\frac{1}{606} - \frac{1}{191}\right)\right\}}$$

$$\Rightarrow \frac{\partial m^*}{\partial v_e} = 0.7845$$

$$\Rightarrow \Delta m^* = 0.7845 \times \Delta V_e$$
$$= 0.7845 \times 10 \times 9.8$$
$$= 76.8 \text{ kg}$$

suppose 2nd stage is improved by 10s (I_{sp})

$$\frac{\partial m^*}{\partial v_e} = \frac{-\ln\left[\frac{606}{191}\right]}{2763\left\{\left[\frac{1}{1886} - \frac{1}{719}\right] + \left[\frac{1}{606} - \frac{1}{191}\right]\right\}}$$

$$= 0.939693$$

$$\Rightarrow \Delta m^* = 0.939693 \times 10 \times 9.8$$
$$= 92.0899 \text{ kg}$$

It is better to make the specific impulse change in second stage.