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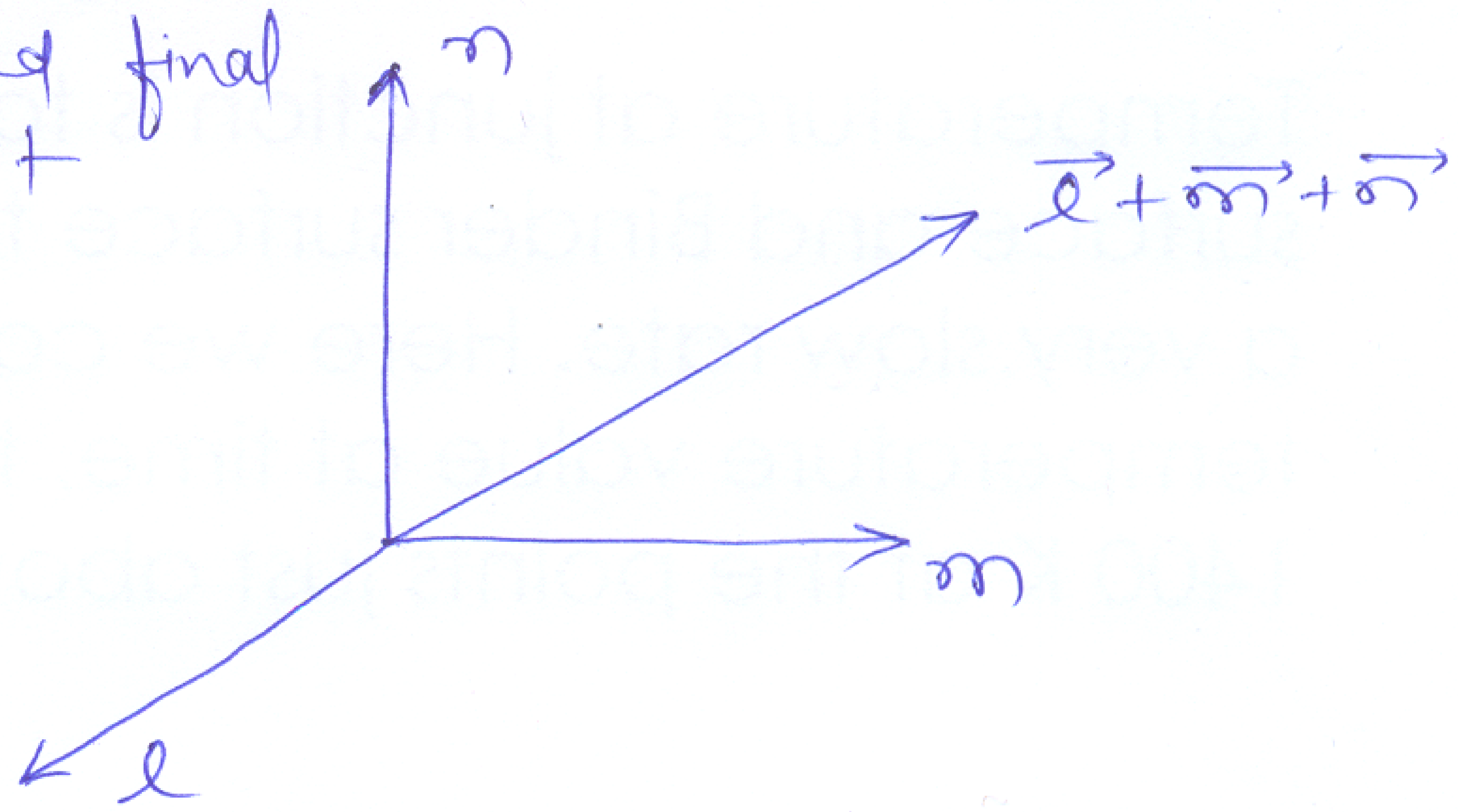
Sol (53.) Let the rotated final frame B be B with unit vectors $\hat{l}', \hat{m}' \,\&\, \hat{n}'$.



Now, Using the result of Q. No. 58,

where

$$\boxed{\Omega t = 60^\circ} \quad \text{and}$$

$\vec{v} = (1,1,1)$ [axis of rotation l, m, n]

then the rotation matrix describing instantaneous orientation of B relative to its initial orientation is

$$R(t) = I + (1-\cos\Omega t)(v\times)^2 + \sin\Omega t\,(v\times)$$

$$\therefore \quad R = I + \left(1-\tfrac{1}{2}\right)\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}^2 + \frac{\sqrt{3}}{2}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} + \frac{\sqrt{3}}{2}\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1/2 & 1/2 \\ 1/2 & -1 & 1/2 \\ 1/2 & 1/2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\sqrt{3}/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 0 & -\sqrt{3}/2 \\ \sqrt{3}/2 & \sqrt{3}/2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right) & \left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right) \\ \left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right) & 0 & \left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right) \\ \left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right) & \left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right) & 0 \end{bmatrix}$$

This R that we have obtained is the rotation matrix which transforms the component of a vector in the rotated frame to the components in the original frame.

Now, to express the unit vectors of the ~~rotated~~ frame B in terms of ~~original~~ l, m, n.

$$(\bar{x})_I = R(\bar{x})_B$$

components of unit vector $\hat{l}'$ of B in I frame will be $= R\begin{bmatrix}1\\0\\0\end{bmatrix}$

$= [\text{first colum of } R]$

$$= \begin{bmatrix}0\\(\sqrt{3}/2+1/2)\\(1/2+\sqrt{3}/2)\end{bmatrix}$$

Similarly $\hat{m}'$ of B in I frame $= R\begin{bmatrix}0\\1\\0\end{bmatrix}$

$$= \begin{bmatrix}(1/2-\sqrt{3}/2)\\0\\(1/2+\sqrt{3}/2)\end{bmatrix}$$

and, $\hat{n}'$ of B in I frame $= R\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$= \begin{bmatrix}(1/2+\sqrt{3}/2)\\(1/2-\sqrt{3}/2)\\0\end{bmatrix}$$

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