BLUE in case of vector measurements:

The previous situation involving the sensor measurements x_1 & x_2 can be recast as: ~~follows~~

$$Z = Hx + v.$$

where $Z = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ – measurement vector.

$H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x =$ qty being measured/estimated.

$V = \begin{bmatrix} \delta x_1 \\ \delta x_2 \end{bmatrix}$ – measurement error.

In general, Z & V – vectors of size l

x – vector of size $n < l$

H – matrix of size $l \times n$.

Assuming the ~~The~~ measurement error to have zero mean & covariance matrix $E(vv^T) = P_{vv}$, Explain.

The best ~~esti~~ linear unbiased estimate of x is given by

$$\hat{x} = (H^T P_{vv}^{-1} H)^{-1} H^T P_{vv}^{-1} z.$$

The covariance matrix of the resulting estimation error is

$$P_{\tilde{x}\tilde{x}} = \left(H^T R^{-1} H\right)^{-1}.$$

$\underline{Ex}$. For $H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $P_{vv} = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1,x_2} \\ \sigma_{x_1,x_2} & \sigma_{x_2}^2 \end{bmatrix}$,

$$P_{vv}^{-1} = \frac{1}{(\sigma_{x_1}^2 \sigma_{x_2}^2 - \sigma_{x_1,x_2}^2)} \begin{bmatrix} \sigma_{x_2}^2 & -\sigma_{x_1,x_2} \\ -\sigma_{x_1,x_2} & \sigma_{x_1}^2 \end{bmatrix}.$$

$$H^T P_{vv}^{-1} H = \frac{\sigma_{x_1}^2 + \sigma_{x_2}^2 - 2\sigma_{x_1,x_2}}{\sigma_{x_1}^2 \sigma_{x_2}^2 - \sigma_{x_1,x_2}^2}$$

$$H^T P_{vv}^{-1} = \frac{1}{(\sigma_{x_1}^2 \sigma_{x_2}^2 - \sigma_{x_1,x_2}^2)} \begin{bmatrix} \sigma_{x_2}^2 - \sigma_{x_1,x_2} & \sigma_{x_1}^2 - \sigma_{x_1,x_2} \end{bmatrix}.$$

$$\therefore \quad (H^T P_{vv}^{-1} H)^{-1} H^T P_{vv}^{-1} = \begin{bmatrix} \dfrac{\sigma_{x_2}^2 - \sigma_{x_1,x_2}}{\sigma_{x_1}^2 + \sigma_{x_2}^2 - 2\sigma_{x_1,x_2}} & \dfrac{\sigma_{x_1}^2 - \sigma_{x_1,x_2}}{\sigma_{x_1}^2 + \sigma_{x_2}^2 - 2\sigma_{x_1}\sigma_{x_2}} \end{bmatrix}.$$

& $(H^T P_{vv}^{-1} H)^{-1} = \dfrac{\sigma_{x_1}^2 \sigma_{x_2}^2 - \sigma_{x_1 x_2}^2}{\sigma_{x_1}^2 + \sigma_{x_2}^2 - \sigma_{x_1 x_2}^2}$ as before.