## Linearized terminal phase analysis of pure pursuit.

Let $\beta_0$ & $R_0$ be the LOS angle & range at time $t_0$. Let $t_f$ be the time of intercept. Assume that $\beta \ll 1$ on $[t_0, t_f]$. Then, the range rate & LOS rate equations become

$$\dot{R} = V_T - V_m$$

$$\dot{\beta} = -\frac{V_T \beta}{R}.$$

$$\therefore R(t) = R_0 + (V_T - V_m)t.$$

$$t_f = R_0 / (V_m - V_T)$$

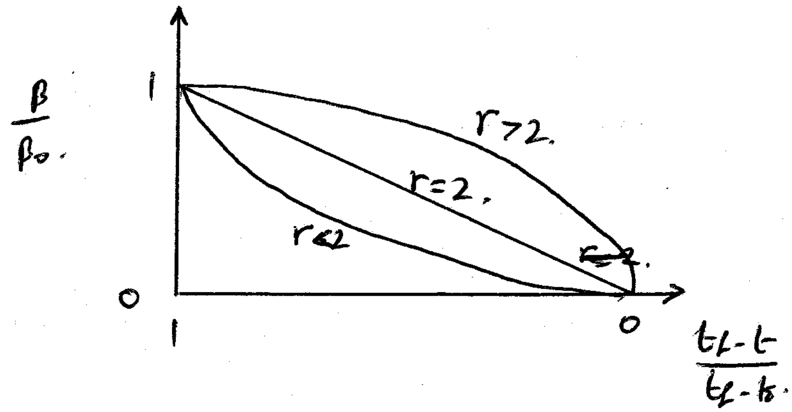
Hence $R(t) = (V_T - V_m)(t - t_f)$

$$\therefore \dot{\beta} = \frac{-V_T \beta}{(V_T - V_m)(t - t_f)} = \frac{-\beta}{(r-1)(t-t_f)}$$

$$\therefore \ln \beta = \frac{1}{r-1} \ln(t - t_f) + \text{Const}.$$

$$\therefore \ln \frac{\beta}{(t - t_f)^{1/r-1}} = \text{Const.} = \ln \frac{\beta_0}{(t_0 - t_f)^{1/r-1}}$$

$$\therefore \beta = \beta_0 \left(\frac{t_f - t}{t_f - t_0}\right)^{1/r-1}.$$




## Linearized miss distance analysis of pursuit guidance.

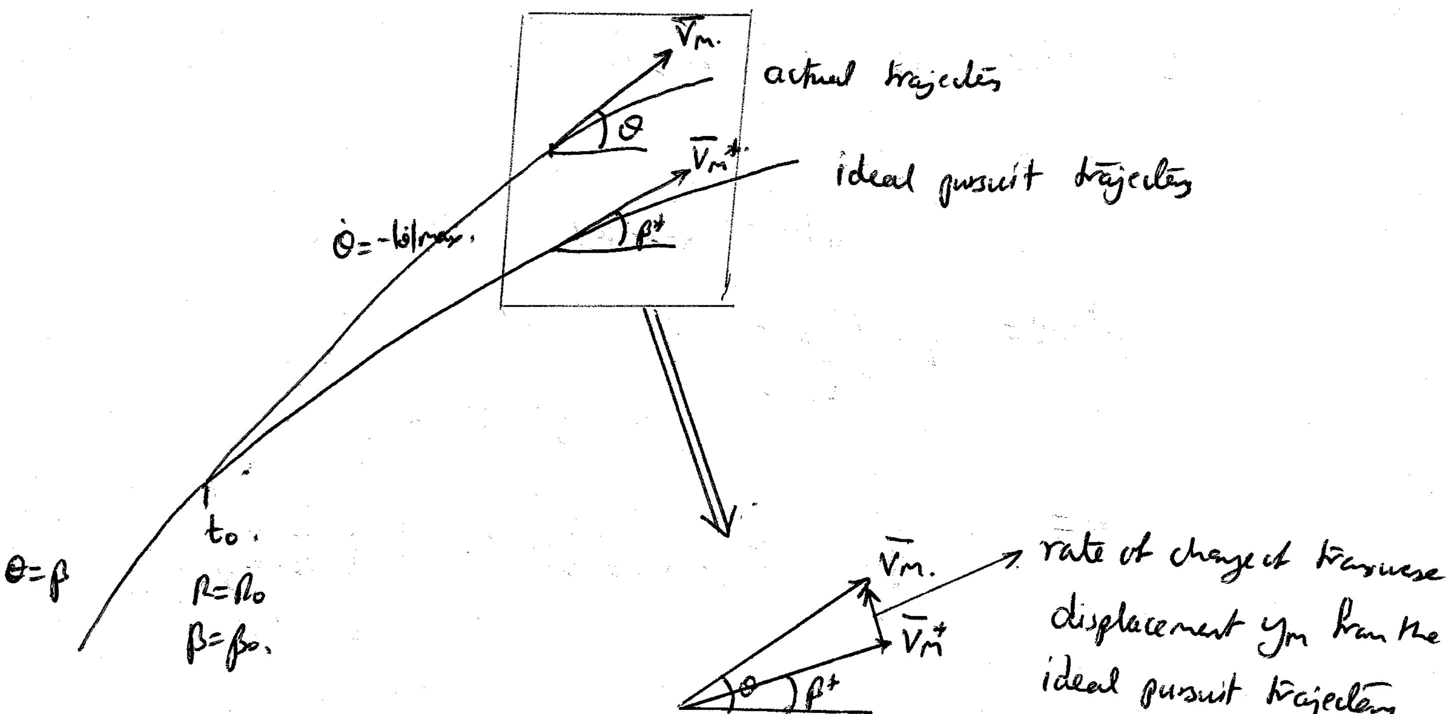
Miss distance analysis looks at the performance of the guidance law when the behaviour of the missile departs from what is ideally assumed. Examples of such departure from the ideal include an upper limit on the lateral acceleration that the missile can generate, & time lags in the flight control system.

Here we analyze the miss distance that results when the LATAX that can be generated by the missile is limited. The limited LATAX results in a lower limit on the radius of turn & an upper limit on the turn rate. We will limit our attention only to the terminal phase. Since the LATAX requirement in the terminal phase is unbounded only in the case where $r>2$, we will assume that $r>2$.

Suppose the missile follows the ideal pursuit trajectory upto time $t_0$, & the LATAX saturation comes into play at $t = t_0$. Let $\beta_0$ & $R_0$ be the ~~range of~~ LOS angle & range at time $t_0$. Let $\beta^*$ denote the LOS along the ideal pursuit trajectory starting at $t_0$. Then

$$\theta(t_0) = \beta^*(t_0) = \beta_0, \qquad \dot{\theta}(t) = -|\dot{\theta}|_{max}, \quad t > t_0.$$

$$\dot{\theta}(t_0) = \dot{\beta}^*(t_0) = -|\dot{\theta}|_{max}.$$




$$\dot{y}_m = V_m(\theta - \beta^*)$$

miss distance $M = y_m(t_f)$, $t_f$ = intercept time for the ideal pursuit trajectory.

Now, for $t > t_0$,

$$\beta^+(t) = \beta_0 \left(\frac{t_f - t}{t_f - t_0}\right)^{1/r-1}.$$

$$\therefore -|\dot{\theta}|_{max} = \dot{\beta}^+(t_0) = \frac{\beta_0}{(1-r).(t_f-t_0)} \left(\frac{t_f - t}{t_f - t_0}\right)^{\frac{2-r}{r-1}} \Bigg|_{t=t_0}$$

$$= \frac{\beta_0}{(1-r)(t_f-t_0)}$$

For $t > t_0$, $\dot{\theta}(t) = -|\dot{\theta}|_{max}$.

$$\therefore \theta(t) = \theta_0 - |\dot{\theta}|_{max}(t - t_0).$$

$$= \beta_0 - \frac{\beta_0}{(1-r)} \cdot \frac{(t-t_0)}{(t_f-t_0)}.$$

$$\beta^+(t) = \beta_0 \left(\frac{t_f - t}{t_f - t_0}\right)^{1/r-1}.$$

$$\therefore \dot{y}_m(t) = V_m \left[\beta_0 - \frac{\beta_0}{1-r}\left(\frac{t-t_0}{t_f-t_0}\right) - \beta_0 \left(\frac{t_f - t}{t_f - t_0}\right)^{1/r-1}\right].$$

On integrating from $y(t_0) = 0$, we get

$$y_m(t) = V_m \Bigg[\beta_0 (t - t_0) - \frac{\beta_0}{2(1-r)} \frac{(t-t_0)^2}{(t_f-t_0)}$$

$$+ \frac{\beta_0}{(t_f-t_0)^{1/r-1}} \cdot \frac{r-1}{r} \left[(t_f - t)^{\frac{r}{r-1}} - (t_f - t_0)^{r/r-1}\right]\Bigg]$$

miss distance

$$M = y_m(t_f)$$

$$= V_m\left[\beta_0 (t_f - t_0) - \frac{\beta_0}{2(1-r)} (t_f - t_0) - \frac{\beta_0}{(t_f - t_0)^{\frac{1}{r-1}}} \cdot \frac{r-1}{r} (t_f - t_0)^{r/r-1}\right]$$

$$= V_m . \beta_0 (t_f - t_0) \left[1 - \frac{1}{2(1-r)} - \frac{r-1}{r}\right].$$

$$= V_m \beta_0 (t_f - t_0) \left[\frac{2-r}{2r(1-r)}\right].$$

Now $t_f - t_0 = \dfrac{R_0}{V_m - V_T}$ for the ideal pursuit trajectory.

$$\therefore M = \frac{V_m \beta_0 R_0}{V_m - V_T} \left[\frac{2-r}{2r(1-r)}\right] = \frac{\beta_0 R_0 . r}{(r-1)} \left[\frac{2-r}{2r(1-r)}\right].$$

$$= \frac{\beta_0 R_0 . (r-2)}{2(r-1)^2}.$$

In general,

$$\dot{R} = V_T \cos(\beta - \theta_T) - V_m \cos(\beta - \theta_m)$$

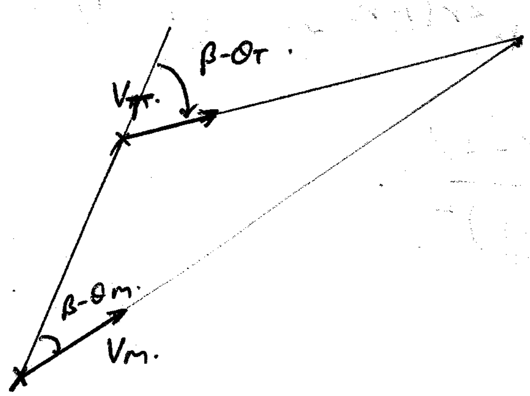
$$\dot{\beta} = -\frac{V_T}{R} \sin(\beta - \theta_T) + \frac{V_m}{R} \sin(\beta - \theta_m)$$

For fixed lead, set $\theta_m = \beta - \theta_L$. $\theta_L$ - Constant lead angle.

The above equations also imply that, for a constant bearing intercept, we must have

$$V_T \sin(\beta - \theta_T) = V_m \sin(\beta - \theta_m)$$

~~This equation~~ In the case where the missile aswell as the target are moving ~~on a~~ along straight lines at constant speeds, the equation above describes the so-called collision triangle.




COLLISION TRIANGLE