

If the missile and the target are on a collision triangle, then $\dot{\beta} = 0$, & hence proportional guidance yields $\dot{\beta}_m = 0$.

$\therefore \beta, \theta_m, \theta_T$ remain constant.

$$\begin{aligned} \therefore \dot{R} &= V_T \cos(\beta - \theta_T) - V_m \cos(\beta - \theta_m) \\ &= -V_c, \end{aligned}$$

$V_c =$ closing velocity, remains constant.

\therefore time to intercept $t_f - t_0 = \frac{V_c}{R_0}$, R_0 - initial range

range-to-go is $R(t) = \frac{R_0(t_f - t)}{(t_f - t_0)} = V_c t_{go}$,

$$t_{go} = t_f - t = \text{time-to-go till intercept.}$$

Note that ~~this~~ a collision triangle requires the missile to be launched at a specific launch angle & since by

$$V_T \sin(\beta - \theta_T) = V_m \sin(\beta - \theta_m).$$

How will the engagement change if the missile has a small initial launch error, or if the target performs a small maneuver?

To answer this question, we will study perturbations about the nominal collision triangle ~~and~~ engagement.

$$\therefore \text{set } R(t) = R^*(t) + \delta R(t), \quad \text{where } R^*(t) = \frac{R_0(t_f - t)}{(t_f - t_0)} = V_c t_{go}.$$

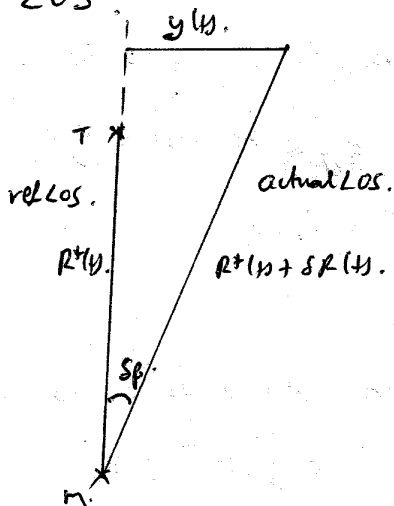
$$\beta(t) = \beta^* + \delta \beta(t), \quad \theta_T(t) = \theta_T^* + \delta \theta_T(t),$$

$$\theta_m(t) = \theta_m^* + \delta \theta_m(t).$$

we have

$$\begin{aligned} \delta \dot{\beta} &= \left[\frac{V_T}{R^{*2}} \sin(\beta^* - \alpha_T^*) - \frac{V_m}{R^{*2}} \sin(\beta^* - \alpha_m^*) \right] \delta R \\ &\quad - \frac{V_T}{R^*} \cos(\beta^* - \alpha_T^*) (\delta \beta - \delta \alpha_T) + \frac{V_m}{R^*} \cos(\beta^* - \alpha_m^*) (\delta \beta - \delta \alpha_m) \\ &= + \frac{V_c}{R^*} \delta \beta + \frac{V_T}{R^*} \cos(\beta^* - \alpha_T^*) \delta \alpha_T - \frac{V_m}{R^*} \cos(\beta^* - \alpha_m^*) \delta \alpha_m \end{aligned}$$

Due to perturbations in missile ~~not~~ & target motions from the ideal collision triangle, the LOS will change from the constant LOS in the collision triangle, which we refer to as the reference LOS. The following figure shows the reference LOS & perturbed LOS.



$y(t)$ = component of relative displacement vector \perp to ref. LOS.

By convention, the miss distance is defined as $y(t_e)$.

The idea is that, if $y(t_e) = 0$, then the LOS ~~rotation~~ at the end of engagement is the same as the reference LOS.

To a first order,
$$\delta \beta(t) \approx \frac{y(t)}{R^*(t)} = \frac{y(t)}{V_c t_{go}}$$

$$\dot{y} = -V_c \delta \beta + R^* \delta \dot{\beta}$$

$$= V_T \cos(\beta^* - \alpha_T^*) \delta \alpha_T - V_m \cos(\beta^* - \alpha_m^*) \delta \alpha_m$$

$$\therefore \ddot{y} = V_T \cos(\beta^* - \alpha_T^*) \delta \dot{\alpha}_T - V_m \cos(\beta^* - \alpha_m^*) \delta \dot{\alpha}_m$$

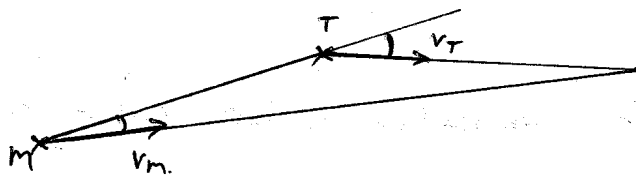
Set $V_T \delta \dot{\theta}_T = \delta n_T$ - perturbation in the target ~~nominal~~ ^{lateral} accn.

$V_m \delta \dot{\theta}_m = \delta n_m$ - perturbation in the missile lateral accn.

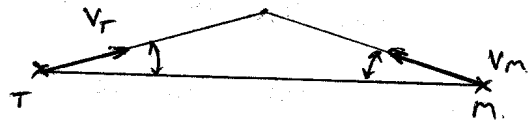
$$\ddot{y} = \cos(\beta^* - \theta_T^*) \delta n_T - \cos(\beta^* - \theta_m^*) \delta n_m.$$

In the special case where the nominal collision triangle represents a near tail chase or near head-on collision, the angles $\beta^* - \theta_T^*$ & $\beta^* - \theta_m^*$ are small, & hence

$$\ddot{y} = \delta n_T - \delta n_m.$$



NEAR TAIL CHASE



NEAR HEAD-ON.

The missile lateral acceleration command issued by the guidance command is

$$\delta n_c = \lambda V_m \delta \beta.$$

The evolution of all perturbations described above can be represented by the following ~~block~~ ^{block} diagram, called the homing loop.