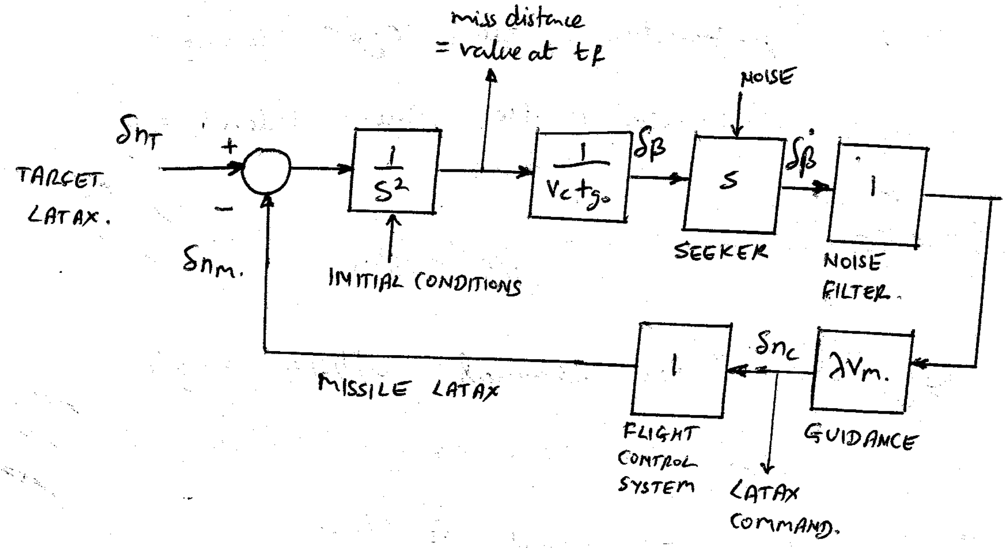


PG HOMING LOOP.

~~Noise~~ Note: The loop contains a time-varying element. ∴ Classical s-domain analysis techniques do not apply. Needs analytical integration or simulations.

Models for the noise filter, seeker noise, flight control system lags can be incorporated to study their effects on the miss distance.

In the absence of target maneuvers assuming perfect flight control, the missile lateral acceleration is

$$\delta\eta_m = \delta n_c = \lambda V_m \delta\dot\beta$$

$$\therefore \ddot y = -\lambda V_m \delta\dot\beta$$

$$\therefore \dot y = -\lambda V_m \delta\beta + C_1$$

$$= -\lambda V_m \frac{y}{V_c t_{go}} + C_1.$$

$$\therefore \dot y + \frac{\lambda V_m y}{V_c (t_f - t)} = C_1$$

The general solution to the equation

$\dot{y} + a(t) y = h(t)$ is

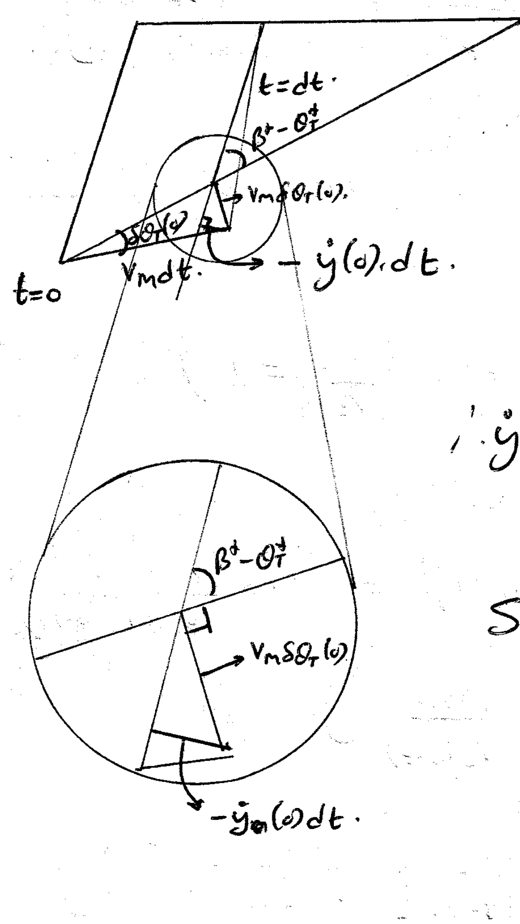
$$y(t) = e^{-\int_0^t a(s)ds}\left[\int_0^t h(\tau).e^{\int_0^\tau a(s)ds} d\tau. + C_2\right].$$

In our case, $a(t) = \frac{\partial V_m}{V_c(t_f - t)}$, $h(t) = C_1$

$$\int_0^t a(s)\,ds = \frac{\partial V_m}{V_c} \ln\left(\frac{t_f}{t_f - t}\right), \quad e^{\int_0^t a(s)ds} = \left(\frac{t_f}{t_f - t}\right)^{\frac{\partial V_m}{V_c}}$$

The initial conditions are given by $y(0) = 0 \Rightarrow C_2 = 0$

To find $\dot{y}(0)$, consider the following diagram:



$\therefore \dot{y}(0) = -V_m \delta\theta_T(0) \cos(\beta^* - \theta_T^*)$.

Since $y(0) = 0$, the eqn. for $\dot{y}$ yields $C_1 = \dot{y}(0)$

$= -V_m \delta\theta_T(0).\cos(\beta^* - \theta_T^*)$.

$$\therefore y(t) = \left(\frac{t_f - t}{t_f}\right)^{\frac{\lambda v_m}{v_c}} \int_0^t C_1 \left(\frac{t_f}{t_f-\tau}\right)^{\frac{\lambda v_m}{v_c}} d\tau.$$

$$= (t_f - t)^{\frac{\lambda v_m}{v_c}} \int_0^t C_1 \, (t_f - \tau)^{-\frac{\lambda v_m}{v_c}} d\tau.$$

$$= (t_f - t)^{\frac{\lambda v_m}{v_c}} \frac{C_1}{1 - \frac{\lambda v_m}{v_c}} \; -(t_f-\tau)^{1-\frac{\lambda v_m}{v_c}} \Big|_0^t, \quad \left(\text{assuming } \frac{\lambda v_m}{v_c} \neq 1\right).$$

$$= (t_f - t)^{\frac{\lambda v_m}{v_c}} \frac{C_1}{1 - \frac{\lambda v_m}{v_c}} \left[t_f^{\,1-\frac{\lambda v_m}{v_c}} - (t_f - t)^{1-\frac{\lambda v_m}{v_c}} \right]$$

$$= \frac{C_1}{1 - \frac{\lambda v_m}{v_c}} \left[t_f \left(\frac{t_f - t}{t_f}\right)^{\frac{\lambda v_m}{v_c}} - (t_f - t) \right].$$

Note that $y(t_f) = 0$. $\therefore$ miss distance due to launch error $= 0$.

(Note: $y(t_f) = 0$ even if $\frac{\lambda v_m}{v_c} = 1$).

The lateral acceleration required is

$$\delta n_m = \delta n_c = \lambda v_m \delta \dot{\beta} = \lambda v_m \frac{d}{dt}\left(\frac{y}{v_c (t_f - t)}\right).$$

$$= \frac{\lambda v_m}{v_c} \frac{y(t)}{(t_f - t)^2} + \frac{\lambda v_m}{v_c (t_f - t)} \dot{y}$$

$$= \frac{\lambda v_m}{v_c} \frac{y}{(t_f - t)^2} + \frac{\lambda v_m}{v_c (t_f - t)} \left[-\frac{\lambda v_m}{v_c (t_f - t)} y + C_1 \right]$$

$$= \frac{\lambda V_m}{V_c} \frac{1}{(t_f-t)^2} \left[1 - \frac{\lambda V_m}{V_c} \right] y + \frac{\lambda V_m}{V_c (t_f - t)} C_1.$$

Substituting for y yields

$$\delta n_m(t) = \frac{\lambda V_m}{V_c (t_f-t)^2} \left[t_f \left(\frac{t_f - t}{t_f} \right)^{\frac{\lambda V_m}{V_c}} - (t_f - t) \right] + \frac{C_1 \lambda V_m}{V_c (t_f - t)},$$

(assuming $\frac{\lambda V_m}{V_c} \neq 1$)

$$= C_1 \frac{\lambda V_m}{V_c t_f} \left(\frac{t_f - t}{t_f} \right)^{\frac{\lambda V_m}{V_c} - 2}$$

~~Note~~: $$= \frac{- \lambda V_m^2 \cos(\beta^* - \theta_T^\alpha)\, \delta\theta_T(0)}{V_c t_f} \left(\frac{t_f - t}{t_f} \right)^{\frac{\lambda V_m}{V_c} - 2}.$$

<u>Note:</u> Initial acceleration increases with guidance constant λ, missile velocity V_m, initial heading error $\delta\theta_T$, & decreases with initial range.

For bounded terminal lateral acceleration, need $\frac{\lambda V_m}{V_c} > 2$.

i.e $\lambda > \frac{2 V_c}{V_m}$. The largest possible value for $\frac{V_c}{V_m}$ is $1 + \frac{V_T}{V_m}$, which is less than 2, assuming $V_T < V_m$.

Q. λ is usually chosen between 3 & 5.