



Let $w(t)$ denote the Laplace inverse of $w(s)$, & let $H(s)$ denote the Laplace transform of $h(t)$.

Then, in the feedback loop, we have

$$h(t) = \frac{1}{t} w * (s-h)(t), \quad * \text{ denotes convolution}$$

$$\therefore th(t) = w * (s-h)(t)$$

$$\therefore \mathcal{L}(th(t)) = -\frac{d}{ds} H(s) = w(s)(1-H(s))$$

$$\therefore \frac{-dH(s)}{1-H(s)} = w(s) ds$$

$$\therefore \mathcal{L}[1-H(s)] = C e^{-\int w(s) ds}$$

Since the signal at (A) has derivative $\dot{y}(w)$ at $t=0$, the signal at (B) must have value 1 at $t=0$. The Laplace transform of the signal at (B) is $\frac{1-H(s)}{s}$. \therefore By the initial value theorem of Laplace transforms —

$$1 = \lim_{s \rightarrow \infty} s \left(\frac{1-H(s)}{s} \right) = \lim_{s \rightarrow \infty} (1-H(s)), \quad \text{that is}$$

$$\lim_{s \rightarrow \infty} H(s) = 0$$

Next, since $w(s) = \frac{\Delta V_m / v_c}{s(1+Ts)} = \frac{\Delta V_m}{v_c} \cdot \frac{1}{s} - \frac{\Delta V_m T}{v_c(1+Ts)}$ 84

we have $1 - H(s) = C e^{\int w(s) ds}$

$$= C e^{\frac{\Delta V_m}{v_c} \left[\ln \frac{s}{1+Ts} \right]} = C \cdot \left[\frac{s}{1+Ts} \right]^{\frac{\Delta V_m}{v_c}}$$

Since $\lim_{s \rightarrow \infty} [1 - H(s)] = 1$, we conclude that $C = T \frac{\Delta V_m}{v_c}$.

$$\therefore 1 - H(s) = \left[\frac{Ts}{1+Ts} \right]^{\frac{\Delta V_m}{v_c}}$$

∴ miss due to step target maneuver,

$$M_T(s) = \frac{\delta n_T}{s^3} \left[\frac{Ts}{1+Ts} \right]^{\frac{\Delta V_m}{v_c}}$$

Can be solved for specific values of $\frac{\Delta V_m}{v_c}$.

For $\frac{\Delta V_m}{v_c} = 4$,

$$M_T(s) = \frac{\delta n_T}{1} \left[\frac{T^4 s}{(1+Ts)^4} \right]$$

$$= \frac{\delta n_T}{1} \frac{s}{(s + 1/T)^4}$$

The inverse Laplace transform is

$$m_T(t) = \frac{\delta n_T}{1} t^2 e^{-t/T} \left(\frac{1}{2} - \frac{t}{6T} \right)$$

yields miss distance as a function of flight time t_F by
on putting $t = t_F$.

The miss distance due to an initial heading error is

$$M_I(s) = \frac{y(\omega)}{s^2} \left[\frac{Ts}{1+Ts} \right]^{\frac{\partial v_m}{v_c}}$$

For $\frac{\partial v_m}{v_c} = 4$, we have

$$M_I(s) = y(\omega) \frac{T^4 s^2}{(1+Ts)^4} = \frac{y(\omega)}{1} \frac{s^2}{(s+1/T)^4}$$

Taking inverse Laplace transform, & substituting

$$y(\omega) = -v_m \delta \theta_m(\omega) \cos(\beta^* - \theta_m^*)$$

$$\approx -v_m \delta \theta_m(\omega) \quad \text{yields}$$

$$M_I(t) = \frac{-v_m \delta \theta_m(\omega)}{1} t e^{-t/T} \left(1 - \frac{t}{T} + \frac{t^2}{6T^2} \right)$$

Substitute $t = t_F$ to obtain miss distance for any flight time.

Note that miss $\rightarrow 0$ as $\frac{t_F}{T} \rightarrow \infty$.