We compare the above ~~ee~~ polar equation with the polar equation

$$r = \frac{a(1-e^2)}{1+e\cos(\theta-\omega_p)} = \frac{a(1-e^2)}{1+e\cos\theta\cos\omega_p + e\sin\theta\sin\omega_p}.$$

of a conic section (circle, ellipse, parabola, hyperbola) having semimajor axis a, eccentricity e and periapsis (~~the~~ point of closest approach) at an angle ω_p (called the argument of perigee).

The comparison yields

$$e\cos\omega_p = \frac{h^2}{\mu r_0} - 1 = \frac{r_0 V_0^2\cos^2\gamma_0}{\mu} - 1$$

& $$e\sin\omega_p = -\frac{h^2}{\mu}\frac{\tan\gamma_0}{r_0} = -\frac{r_0 V_0^2\cos\gamma_0\sin\gamma_0}{\mu}.$$

Hence we get $$e = \sqrt{1+\cos^2\gamma_0\,\frac{r_0V_0^2}{\mu}\left(\frac{r_0V_0^2}{\mu}-2\right)}$$

The orbit is circular if $e=0$, which is possible if & only if $\cos^2\gamma_0 = 1$ & $\frac{r_0V_0^2}{\mu} = 1$.

The orbit is elliptical if $0<e<1$ which is possible ~~if &~~ only if $\frac{r_0V_0^2}{\mu} < 2$.

The orbit is parabolic if $e = \cancel{0}\,1$. $\left(\frac{r_0V_0^2}{\mu} = 2\right)$

& hyperbolic if $e>0$ $\left(\frac{r_0V_0^2}{\mu} > 2\right)$.

~~Semi~~major axis $a = \dfrac{h^2}{\mu(1-e^2)} = \dfrac{r_0}{2 - \dfrac{r_0 V_0^2}{\mu}}$

For a missile, it makes more sense to consider the apoapsis/apogee (~~point of~~ farthest point from earth). ~~The~~ For an elliptical orbit, argument of apogee

$$\omega_a = \omega_p + \pi .$$

∴ The polar equation becomes

$$r = \frac{a(1-e^2)}{1 - e\cos(\theta - \omega_a)}$$

while $\cos\omega_a = -\cos\omega_p = \dfrac{1}{e}\left(1 - \dfrac{r_0 V_0^2}{\mu}\cos^2\gamma_0\right)$

& $\sin\omega_a = -\sin\omega_p = \dfrac{1}{e}\left(+\dfrac{r_0 V_0^2}{\mu}\sin\gamma_0\cos\gamma_0\right)$

Apogee distance $r_a = r(\theta = \omega_a) = \dfrac{a(1-e^2)}{1-e} = a(1+e).$

Maximum altitude $= r_a - R_e.$

Note that the polar equation remains the same even if a different reference axis is chosen for the polar ~~axis~~ coordinate system.