Perigee distance $r_p = r(\theta = \omega_f) = \dfrac{a(1-e^2)}{1+e} = a(1-e)$.

For a missile, $r_p < R_e$.

Hit equation: Gives combinations of initial conditions which lead to a hit at a given target.

~~For a hit~~, To hit a target at an angular location θ_f, set $r(\theta_f) = R_e$, the radius of earth.

Thus
$$R_e = \frac{h^2/\mu}{1 - \frac{h^2}{\mu r_0}\tan\gamma_0 \sin\theta_f + \left(\frac{h^2}{\mu r_0} - 1\right)\cos\theta_f}.$$

Solving for h yields
$$\frac{h^2}{\mu r_0} = \frac{-(\cos\theta_f - 1)}{\frac{r_0}{R_e} - \frac{1}{\cos\gamma_0}\cos(\theta_f + \gamma_0)}$$

i.e
$$\frac{r_0 V_0^2 \cos^2\gamma_0}{\mu} = \frac{-(\cos\theta_f - 1)}{\frac{r_0}{R_e} - \frac{1}{\cos\gamma_0}\cos(\theta_f+\gamma_0)}.$$

$$\therefore \quad V_0 = \boxed{\sqrt{\frac{\mu(1-\cos\theta_f)}{r_0\cos\gamma_0\left[\frac{r_0\cos\gamma_0}{R_e} - \cos(\theta_f+\gamma_0)\right]}}}$$

The equation above, called the hit equation, gives the speed required at $r = r_0$, $\theta = \theta_0 = 0$ to hit a target located at $r = R_e$, $\theta = \theta_F$, for an initial flight path angle γ_0.

To find the time of flight, we recall that

$$r^2 \dot{\theta} = h$$

Hence $$dt = \frac{r^2 d\theta}{h}.$$

$$\therefore \; t_F - t_0 = \frac{1}{h}\int_{\theta_0}^{\theta_F} r^2 d\theta = \frac{1}{h}\int_{\theta_0}^{\theta_F} \frac{(h^2/\mu)^2}{[1 - e\cos(\theta - \omega_a)]^2}\, d\theta$$

Using $$\int \frac{dx}{[a + b\cos x]^2} = \frac{b\sin x}{(b^2 - a^2)(a + b\cos x)} + \frac{2a}{(a^2 - b^2)^{3/2}} \tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\, \tan\frac{x}{2}\right],$$

(see http://integrals.wolfram.com/index.jsp)

we get $$t_F - t_0 = \frac{h^3}{\mu^2}\left[\frac{e\sin(\theta - \omega_a)}{(1 - e^2)(\cancel{a}1 - e\cos(\theta - \omega_a))} + \frac{2}{(1 - e^2)^{3/2}} \tan^{-1}\left[\sqrt{\frac{1+e}{1-e}}\, \tan\frac{(\theta - \omega_a)}{2}\right]\right]_{\theta_0}^{\theta_F}.$$

The hit equation indicates that there ~~could be~~ are several trajectories passing through the current location & the target. These trajectories have different initial velocity vectors (that is, V_0 & $\dot{r}_0$), & different flight times. We can pick a unique trajectory by specifying the flight time. This gives rise to Lambert's problem.