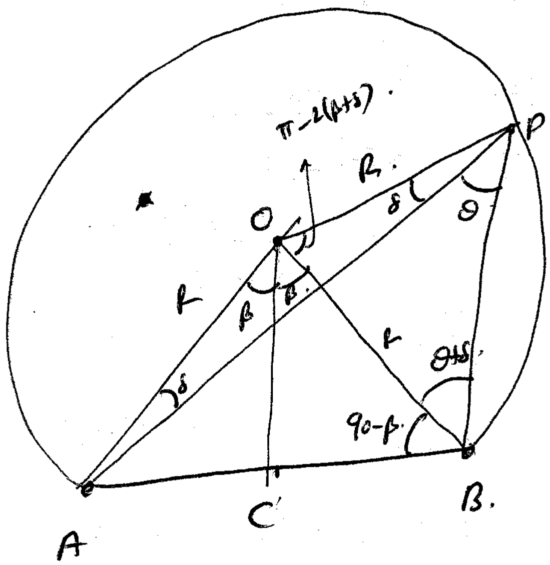
Position fixing using difference between bearings. (Hoffman, Anderson)



Useful when bearing measurements are relative, but the vessel heading is uncertain or unknown.

Consider the circle passing through AB, B & P:

If O is the center of the circle, & θ is the diff. in bearings, i.e. the angle subtended by AB at P.

Let $\angle OAP = \delta$ & $\angle COB = \beta$.

Then $\angle AOC = \beta$, & $\angle OPA = \delta$. — Isoceles Δ's.

~~In the~~ $\angle OBP = \angle OPB = \theta + \delta$. — Isoceles Δ's.

$\angle BOP = \pi - \angle AOB - \angle OAP - \angle OPA = \pi - 2(\beta+\delta)$.

In the Δ OPB, $\pi - 2(\beta+\delta) + 2(\theta+\delta) = \pi \therefore \theta = \beta$.

It follows that all points lying on the circular arc OPB measure the same value θ for the diff. in bearing. Thus the LOP is a circular arc.

To write down the equation, let $c = \frac{r_A + r_B}{2}$,

e – unit vector $\perp$ to $r_A - r_B$.

Radius of the circle $R = \dfrac{\|r_A - r_B\|}{2\cos\theta}$.

P.v. of the center $O = c + \dfrac{\|r_A - r_B\|}{2\tan\theta} e$

The equation is $(r - o)^T (r - o) = \dfrac{\|r_A - r_B\|^2}{4\cos^2\theta}$.

Need atleast 3 bearing measurements to get a fix.

To plot the position fix, a stationpointer is used. Consists of ~~3 arm~~ a protractor with 3 movable arms that can be set to the necessary angles. The protractor is set & moved about over the chart till the 3 arms pass thru the 3 objects whose bearings were used.

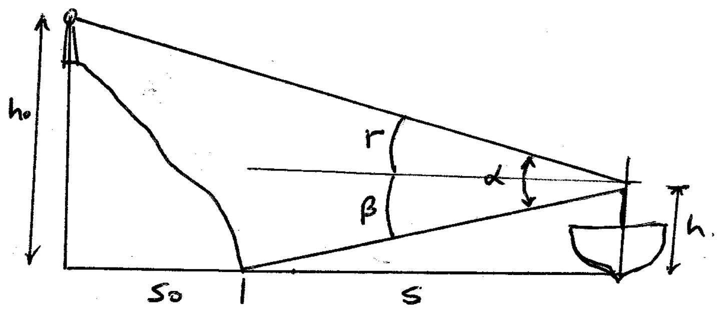
Indirect Ranging: (Hofmann).

Consider a vessel at an unknown distance s from the coast. The sailor measures the vertical angle between a landmark of known height h_0 & the coast using a sextant.

If the sailor is at a height h and the landmark is located at distance s_0 from the coast, then

$$\tan\beta = h/s \quad \tan\gamma = \frac{h_0 - h}{s + s_0}.$$

$$\therefore s = \cot\gamma\,(h_0 - h) - s_0 = \cot(\alpha - \tan^{-1} h/s)(h_0 - h) - s_0$$



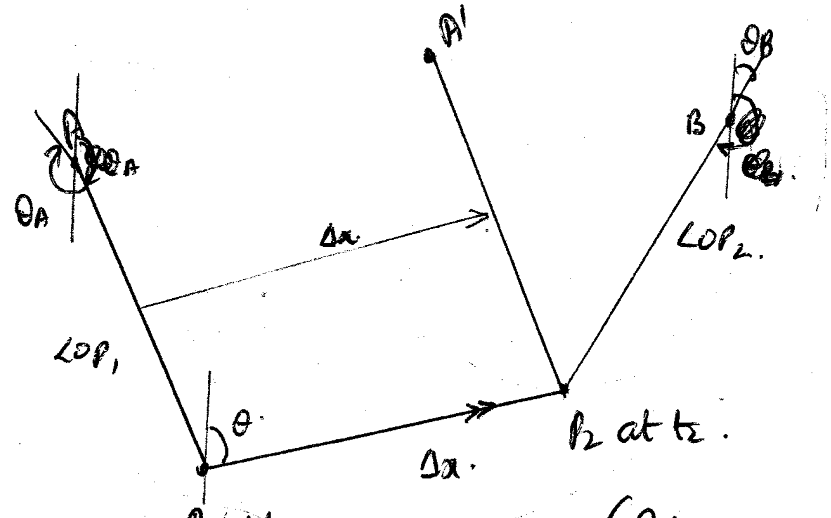
Non simultaneous observations: (Hoffman).

So far we have assumed that the required multiple observations were ~~us~~ obtained simultaneously. However, for higher accuracy, especially in high speed applications, it is necessary to take into account the time elapsed between the measurements. This is ~~so~~ called taking a "running fix", & is possible if the motion of the vehicle/vessel between measurements can be estimated.

Consider a vessel taking a running fix by measuring the true bearings of known points A & B, at instants t_1 & t_2. Suppose the vessel maintains a speed u & a true course α between t_1 & t_2.

Then the displacement of the vehicle in the interval $[t_1, t_2]$ is

$$\Delta x = u \begin{bmatrix} \cancel{co}\ \sin\alpha \\ \cos\alpha \end{bmatrix}.$$



(Advancing the LOP.)

The position fix at P_2 can be obtained by advancing the LOP obtained ~~by~~ from the 1st measurement by an amount Δx. Thus the equations to be solved are

$$\begin{bmatrix} \cos\theta_A & -\sin\theta_A \\ \cos\theta_B & -\sin\theta_B \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} [\cos\theta_A \ \ -\sin\theta_A]^T (r_A + \Delta x) \\ [\cos\theta_B \ \ -\sin\theta_B]^T r_B \end{bmatrix}.$$

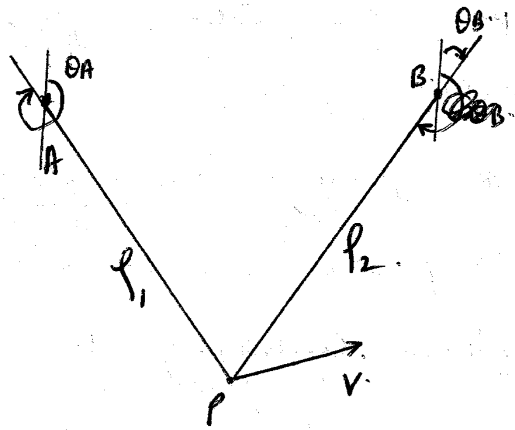
where (x_1, x_2) = coordinates of P_2..

~~Velocity ~~measurement~~ estimation using range-rate measurement. (Hoffman~~

The idea of advancing the LOP applies to any other means of fixing the position, ~~& involves~~. Operationally, it involves replacing r_A by $r_A + \Delta x$, where A is the point used to generate the earlier LOP.

Velocity ~~measurement~~ estimation using range rate measurement.

The observer measures rate of change of range to two stationary objects.



$$\dot\rho_1 = \frac{d}{dt}\sqrt{(r_p-r_A)^T(r_p-r_A)} = \frac{V^T(r_p-r_A)}{\underbrace{\sqrt{(r_p-r_B)^T(r_p-r_A)}}_{\text{unit vector from A to P}}} = \begin{bmatrix}\sin\theta_A & \cos\theta_A\end{bmatrix} V$$

∴ The equation to be solved for V is

$$(-1)? \quad \underbrace{\begin{bmatrix}\sin\theta_A & \cos\theta_A \\ \sin\theta_B & \cos\theta_B\end{bmatrix}}_{\det = \sin(\theta_A-\theta_B)} V = \underbrace{\begin{bmatrix}\dot\rho_1 \\ \dot\rho_2\end{bmatrix}}_{\text{measurement}}$$

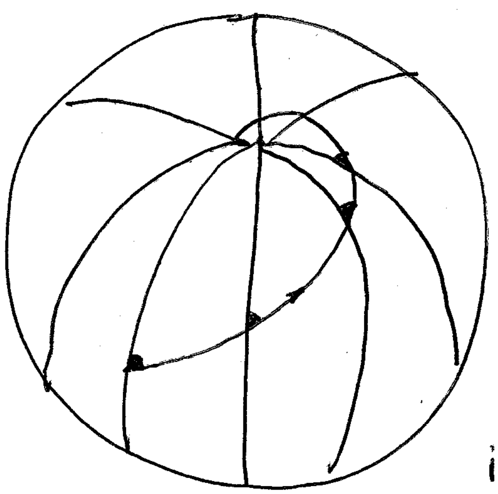
Range rate is measured using Doppler measurements –

$$\Delta f = f_r - f_e = -\frac{f_e 2\dot\rho}{c}$$

c = speed of signal, f_r = freq. received after reflection, f_e = freq. emitted, $\dot\rho$ = range rate. (NOTE : EXTRA FACTOR OF 2)

Courses: (Hoffmann, Anderson, Gardner & Creelman,

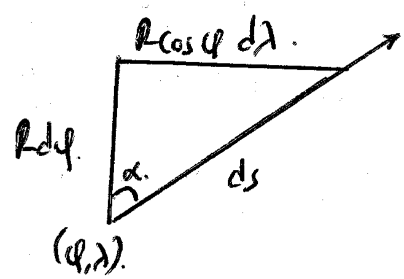
Till recently, ships navigated from point to point by maintaining a constant course (i.e angle w.r.t true north) since this was easy to do. The resulting path, which cuts all meridians at the same angle, is called a rhumb line or loxodrome.



Parallels of latitude & meridians of longitude are special cases of a loxodrome, corresponding to a course of 90 or 270°, & 0 or 180°, respectively.

In the 16th century, Mercator invented the Mercator map. A Mercator map is made by ~~project~~ conformally projecting a sphere (with two poles removed) on a cylindrical surface. Meridians appear as equally spaced parallel vertical lines, while latitudes appear as parallel, but unequally spaced horizontal lines. Rhumb lines, which must cut meridians at a constant angle, appear as straight lines. This property makes them useful for determining the course required to ~~However, Me~~ navigate along a rhumb line between two points.

Equation of a rhumb line &



The figure shows an elemental length ds of a rhumb line at latitude φ & longitude λ. We have

$$\tan\alpha = \frac{R\cos\varphi}{R}\frac{d\lambda}{d\varphi} = \cos\varphi\frac{d\lambda}{d\varphi}$$

$\therefore\ d\lambda = \tan\alpha \dfrac{d\varphi}{\cos\varphi}$. Since $\alpha =$ constant, we have

$$\lambda_B - \lambda_A = \tan\alpha\ \ln\left[\frac{\tan(\pi/4 + \varphi_B/2)}{\tan(\pi/4+\varphi_A/2)}\right].$$

This equation yields the constant course required to navigate ~~bet~~ from A to B.

To find the distance along the rhumb line,

$$ds = R\sqrt{d\varphi^2 + \cos^2\varphi\, d\lambda^2} = R\sqrt{1+\cos^2\varphi\left(\frac{d\lambda}{d\varphi}\right)^2}\,d\varphi$$

$$= R\sqrt{1+\tan^2\alpha}\ d\varphi = R\sec\alpha\, d\varphi$$

distance ~~between A & B~~ along the rhumb line between A & B $= R\sec\alpha\,|\varphi_B - \varphi_A|$ if $\alpha \neq 90°$.

If $\alpha = 90°$, then $\varphi =$ constant, & hence

distance $= R\cos\varphi_A\,(\lambda_B - \lambda_A)$.