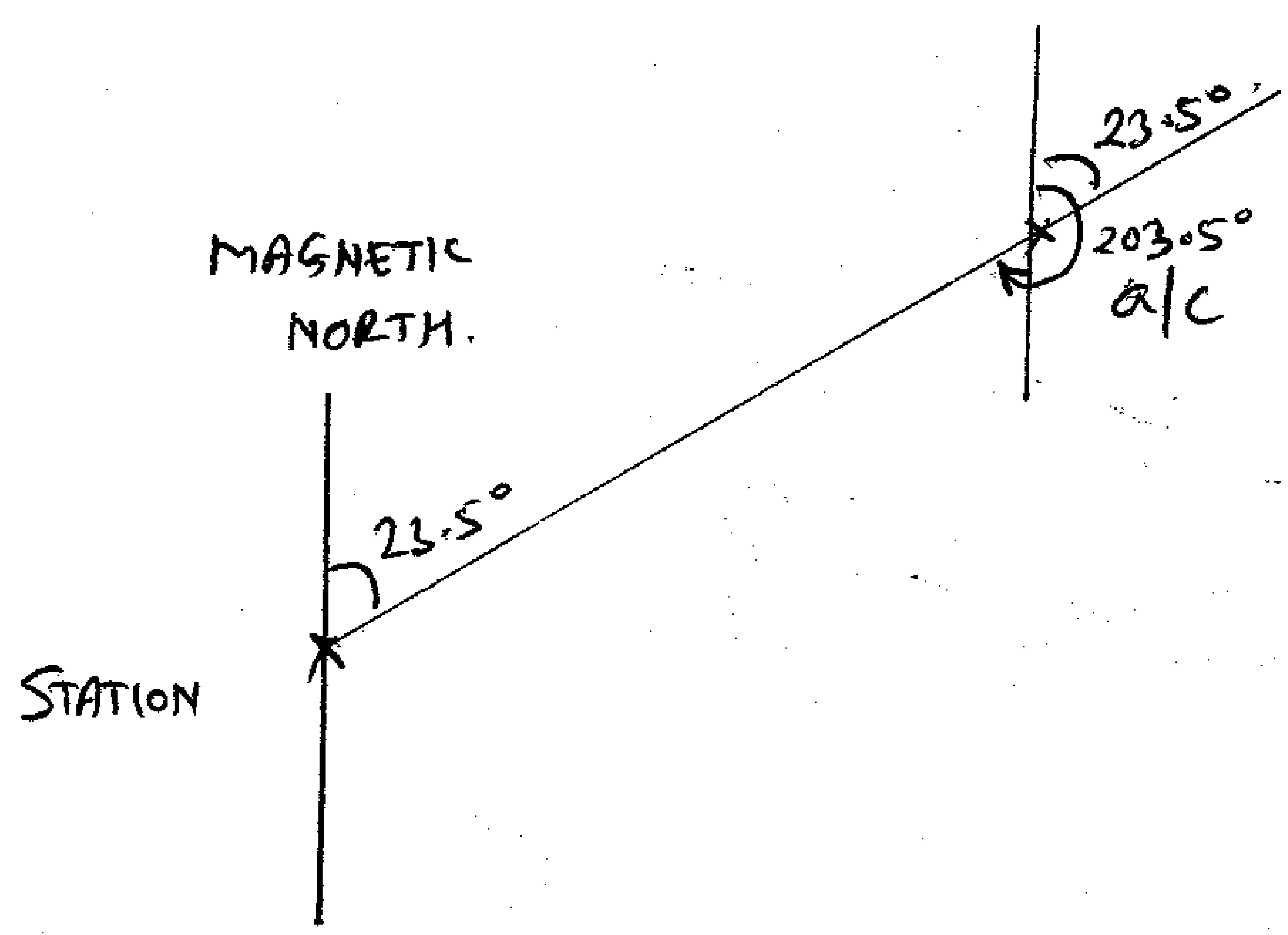
AE457/641 Aut. 2007. Quiz 1. 29/8/07

Solutions.

1). The phase lag between the AM & FM signal directly gives the magnetic bearing of the a/c relative to the station. Hence the bearing of the a/c rel. to the station is 23.5°. Therefore, the bearing of the station relative to the a/c is $180 + 23.5° = 203.5°$



1 mark

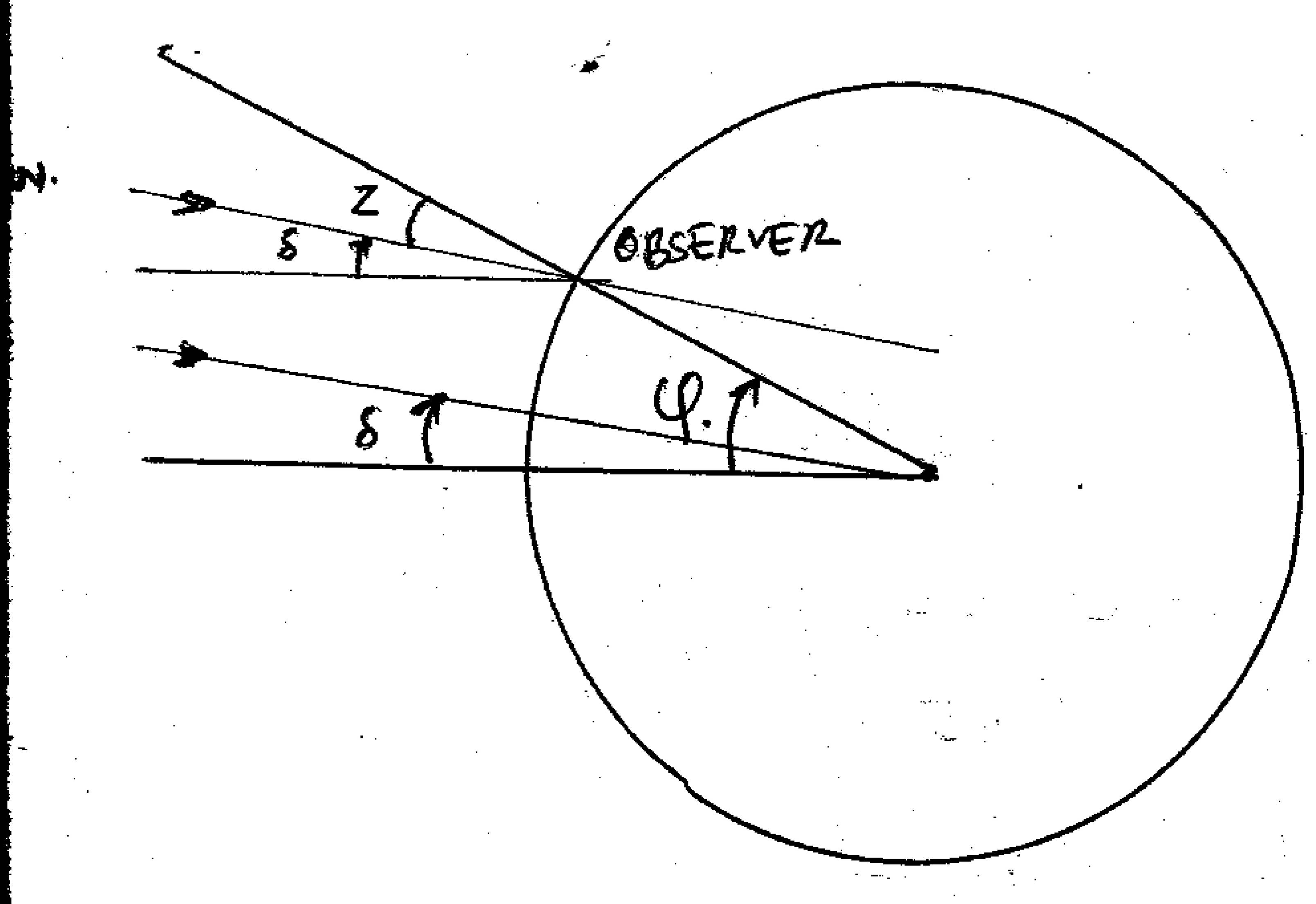
2). The maximum elevation occurs when the sun is overhead the same meridian as the observer. At that time, the ~~geometry is as shown below.~~ latitude directly below the sun is the declination of the sun.

Hence $\delta = 9.24°$. 1/2 mark

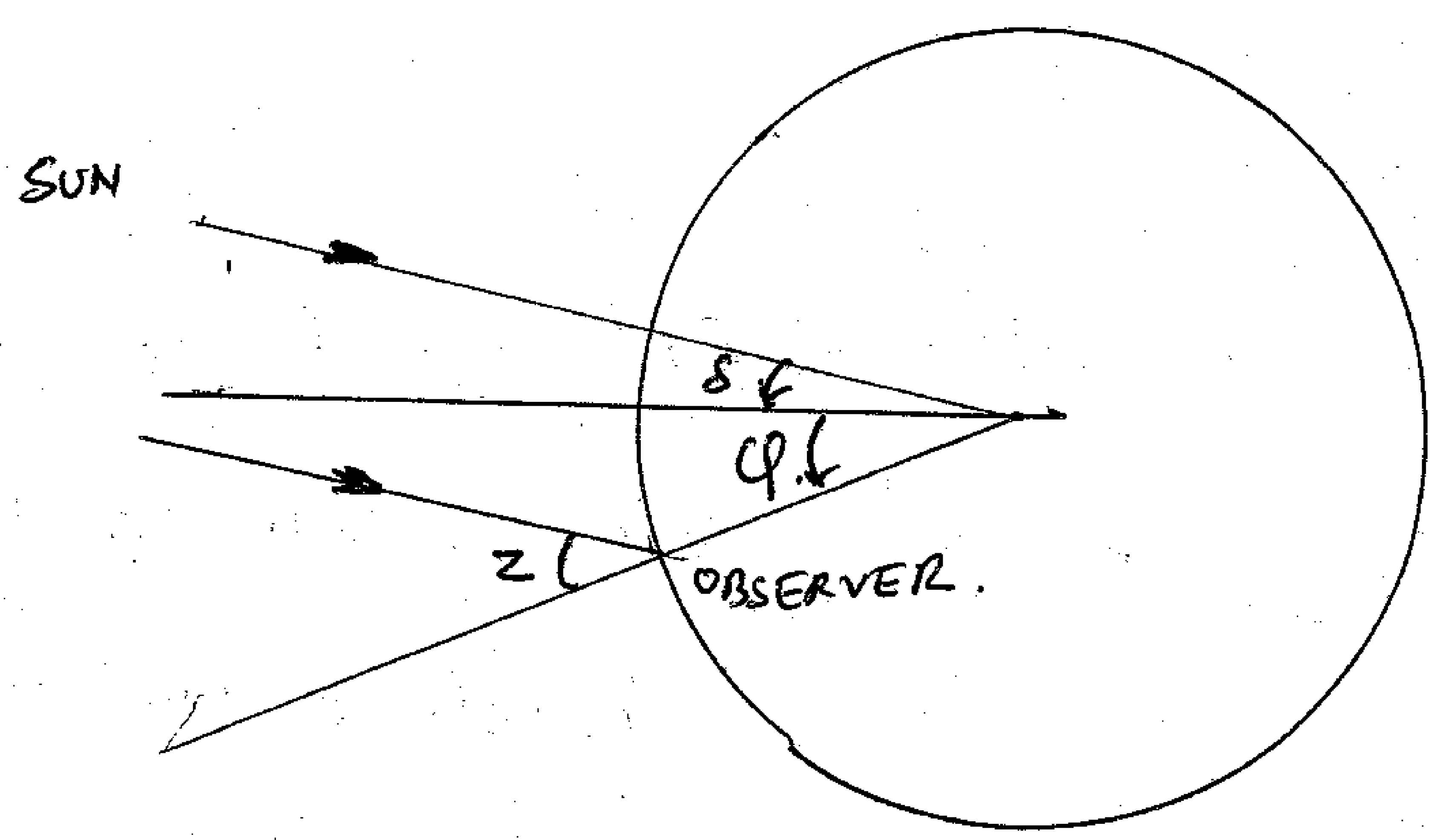
Zenith angle ~~at~~ seen by the observer

$z = 90° - \text{elevation} = 90° - 33°23'$ 1/2 mark
$= 56°37'$

$\therefore$ latitude $\varphi = z + \delta = 66°1'$ N. 1 mark



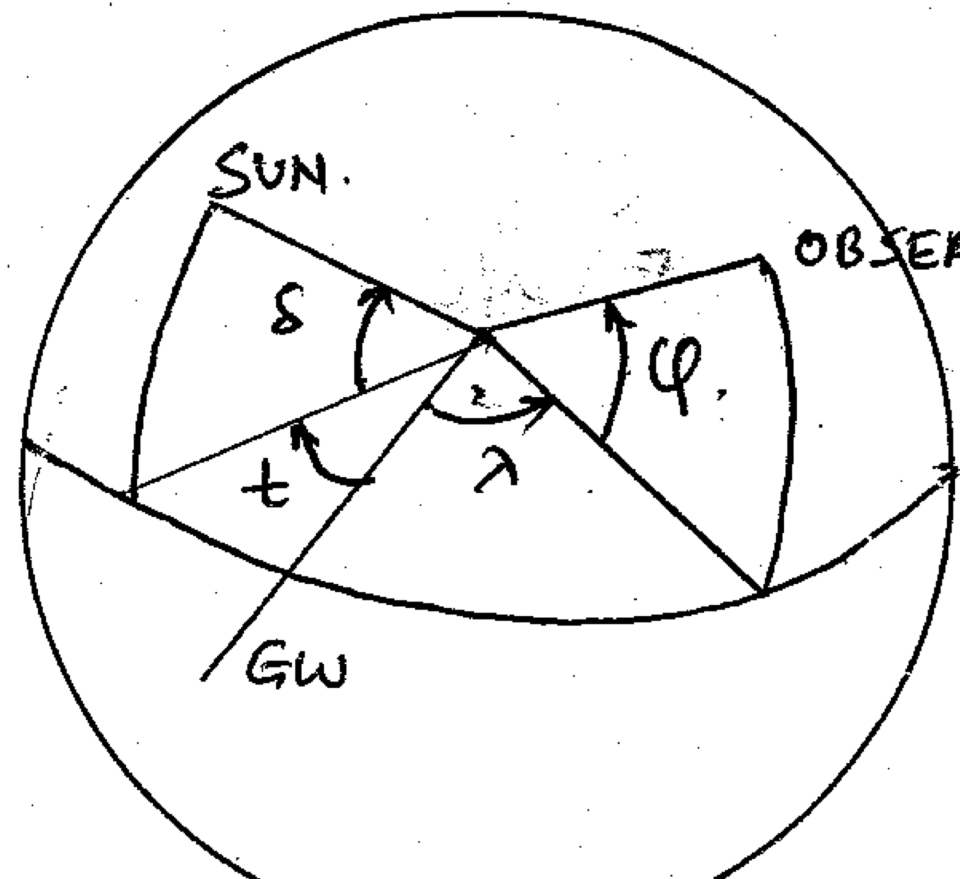
There is another solution possible as shown below.



In this case, the observer lies in the southern hemisphere at a latitude $\varphi = z - \delta = 47^\circ 13'$ S.

Any one solution is acceptable.

3).



Latitude of observer $\varphi = 18^\circ 56' = 18.018^\circ$

Longitude of observer $\lambda = 72^\circ 51' = 72.85^\circ$

Elevation of sun (from previous problem) $\delta = 9^\circ 24' = 9.4^\circ$

GHA of sun $t = 269^\circ 43' = 269.717^\circ$

The zenith vector at the observer is

$$r_0 = \begin{bmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\sin\lambda \\ \sin\varphi \end{bmatrix} = \begin{bmatrix} 0.28 \\ 0.908 \\ 0.309 \end{bmatrix}.$$

1 mark

Unit vector to Sun $r_3 = \begin{bmatrix} \cos\delta\cos t \\ -\cos\delta\sin t \\ \sin\delta \end{bmatrix} = \begin{bmatrix} -0.0048 \\ 0.9865 \\ 0.1633 \end{bmatrix}.$

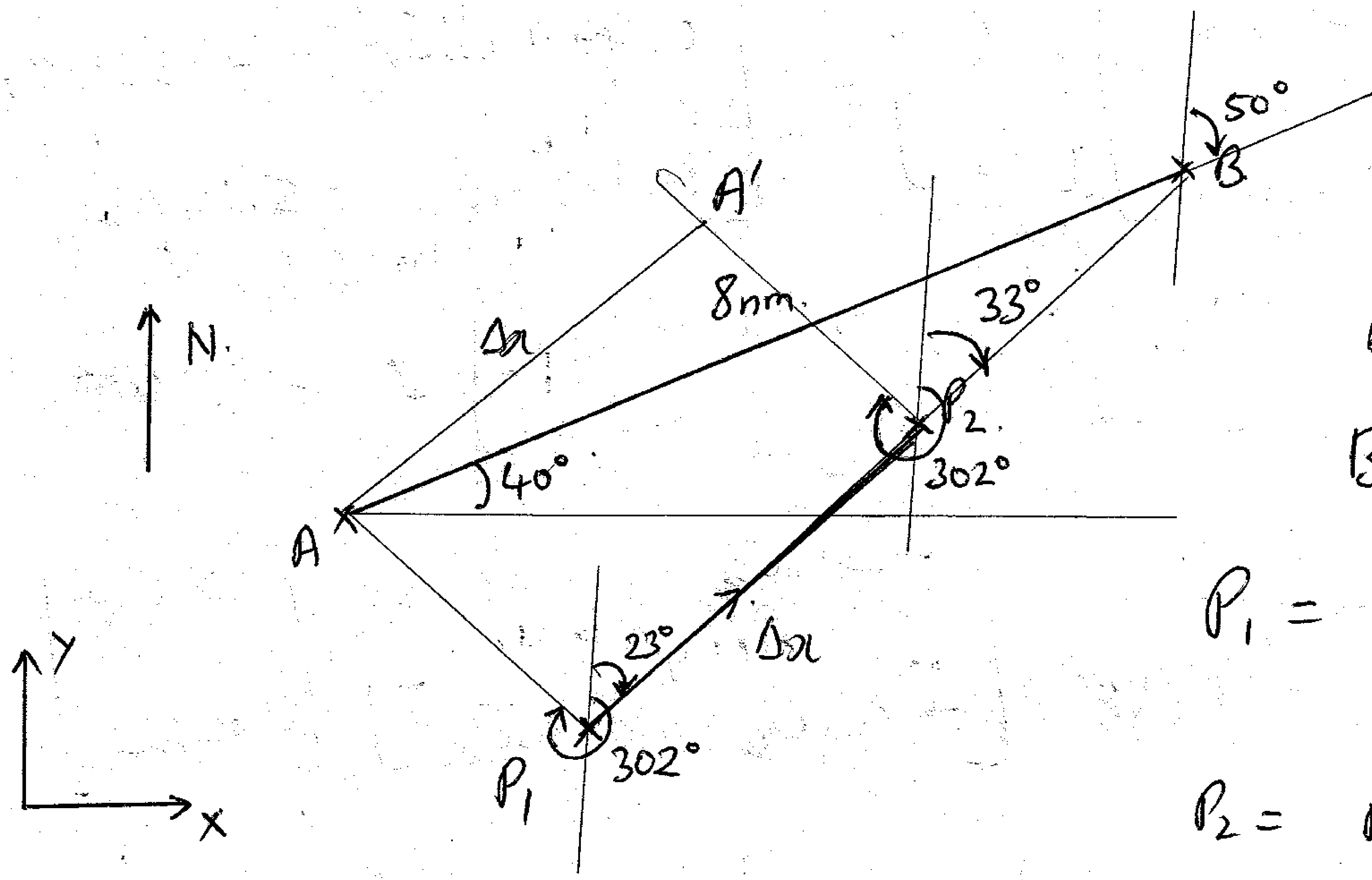
1 mark

. zenith angle is given by $z = \cos^{-1} r_0^T r_3 = 0.945 \therefore z = 19.11^\circ$

$\therefore$ elevation $= 90 - 19.11^\circ = 70.88^\circ$.

1 mark

4.



1 mark for diagram.

$A = (0, 0)$.

$B = (8\sin 50°,\ 8\cos 50°)$
$= (6.12,\ 5.14)$. ½ mark.

P_1 = position at 1st observation.

P_2 = position at 2nd observation.

length ~~Vector~~ . P_1P_2 = ~~Δx~~ = distance travelled in 30min.

$= 2.5$ nm.

∴ vector $P_1P_2 = \Delta x = (2.5\sin 23°\ \ 2.5\cos 23°)$
$= (0.977,\ 2.3)$ nm.

The position at P_2 can be found by advancing the LOP obtained by the 1st observation from P_1 to P_2. This corresponds to taking a bearing measurement to point A', whose position vector relative to A is Δx.

Thus $A' = (0.977,\ 2.3)$. } 1 mark.

The bearing measurement to A' & B are

$\theta_{A'} = 302°$, $\theta_B = 33°$.

If $P_2 = (x_1, x_2)$ ~~the~~ then,

$$\begin{bmatrix} \cos\theta_{A'} & -\sin\theta_{A'} \\ \cos\theta_B & -\sin\theta_B \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.977\cos\theta_{A'} - 2.3\sin\theta_{A'} \\ 6.12\cos\theta_B - 5.142\sin\theta_B \end{bmatrix}$$

$$= \begin{bmatrix} 2.468 \\ 2.332 \end{bmatrix}$$ 1 mark

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\sin(\theta_{A'}-\theta_B)} \begin{bmatrix} -\sin\theta_B & \sin\theta_{A'} \\ -\cos\theta_B & \cos\theta_{A'} \end{bmatrix} \begin{bmatrix} 2.468 \\ 2.332 \end{bmatrix}.$$

$$= \begin{bmatrix} 3.322 \\ 0.8341 \end{bmatrix}.$$

$\therefore$ At the time of the second observation, the ship is 3.322 nm east & 0.8341 nm north of A. $\frac{1}{2}$ mark

Equivalently it is 3.43 nm from A at a bearing (relative to A) of $\tan^{-1}\frac{3.322}{0.8341} = 75.9°$.