Test 2, 24/10/07, SOLUTIONS.

1). It is given that $r = 1.5$.

The closing velocity along an ideal pursuit trajectory is given by

$$V_c = V_m - V_T \cos\beta.$$

$$\therefore \frac{V_c}{V_m} = 1 - \frac{\cos\beta}{r} \qquad | \text{ mark}$$

In the 1st instance, $V_c = V_m$, which yields $\cos\beta_1 = 0$

i.e $\beta_1 = \pi/2$. Thus $\beta_1 = \pi/2$ when ~~range~~ $R = R_1$. 1/2 mark

In the second instance, $V_c = \frac{V_m}{2}$.

$$\therefore 1 - \frac{\cos\beta_2}{1.5} = \frac{1}{2} \quad \text{i.e} \quad \cos\beta_2 = \frac{1.5}{2} = 0.75 \qquad 1/2 \text{ mark}$$

which yields $\beta_2 = 41.4°$.

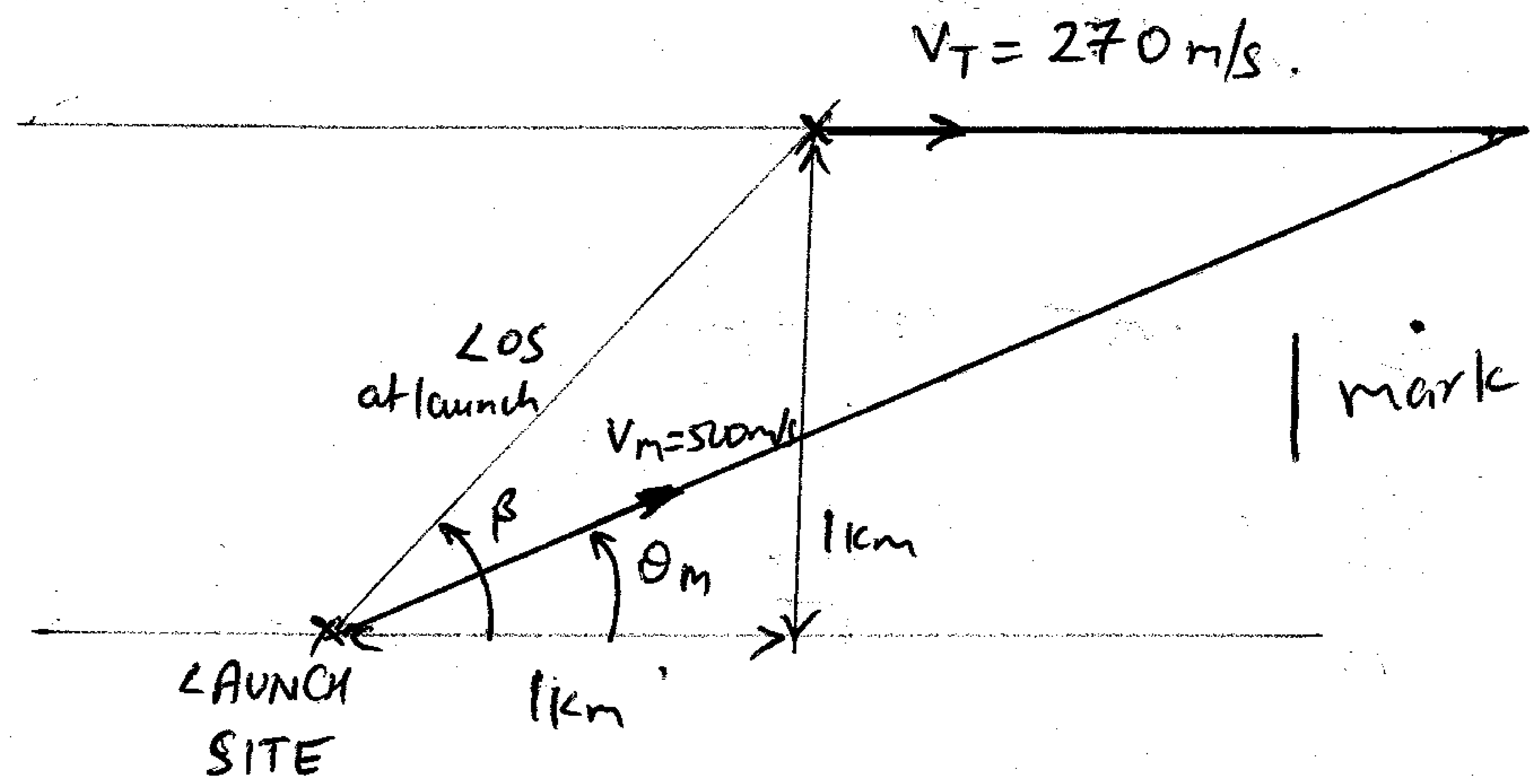
II. For the ideal pursuit trajectory,

$$\frac{R_1 \sin\beta_1}{(\tan\beta_1/2)^r} = \frac{R_2 \sin\beta_2}{(\tan\beta_2/2)^r}.$$

$$\therefore R_2 = \frac{R_1 (\tan\beta_2/2)^r}{\sin\beta_2} = \frac{R_1 (\tan 20.7)^{1.5}}{\sin 41.4} = 0.3512.$$

$$\therefore R_2 = 0.3512 R_1 \qquad | \text{ mark}.$$

2). The information given in the problem can be used to draw the following figure.



1 mark

~~$\beta = 45°$~~ From the given data, $\beta = 45°$, $V_T = 270$ m/s, $\theta_T = 0$

$V_M = 500$ m/s.

For a collision triangle, the LOS rate should be zero. Equivalently, the components of the missile ~~veloci~~ & target velocities perpendicular to the LOS should be equal.

Hence $\quad V_T \sin(\beta - \theta_T) = V_M \sin(\beta - \theta_M)$. 1/2 mark

i.e $\quad 270 \sin 45 = 500 \sin(45 - \theta_M)$.

$$\therefore \theta_M = -\sin^{-1}\left(\frac{270}{500\sqrt{2}}\right) + 45° = 22.55°.$$ 1/2 mark

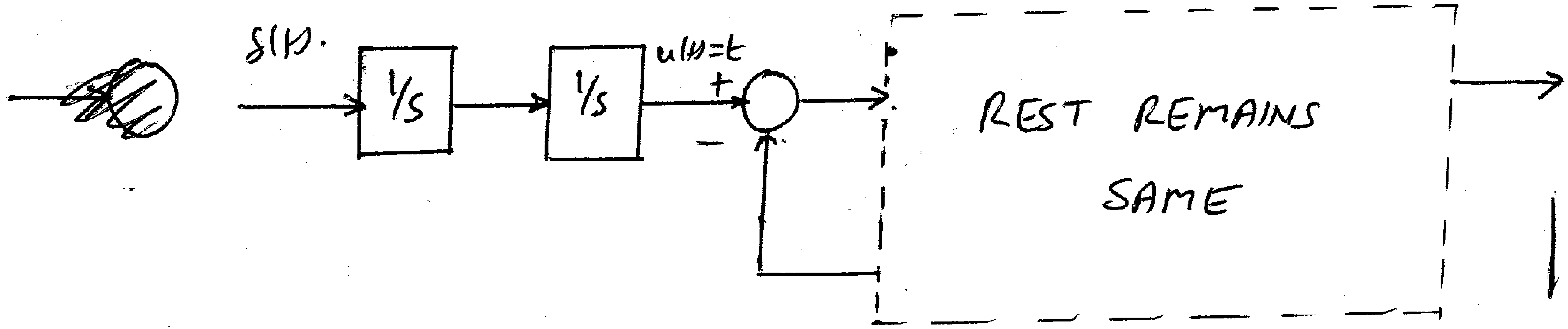
The closing velocity is $\quad -\dot{R} = V_M \cos(\beta - \theta_M) - V_T \cos(\beta - \theta_T)$

$= 500 \cos(45 - 22.55) - 270 \cos 45° = 271.19$ m/s. 1 mark

Initial range $\quad R_0 = \sqrt{2}$ km $= 1414.21$ m.

$\therefore$ time to intercept $= \dfrac{R_0}{V_c} = 5.21$ sec. 1 mark

3. First, redraw the original block diagram to replace all inputs by impulses, and include initial conditions.

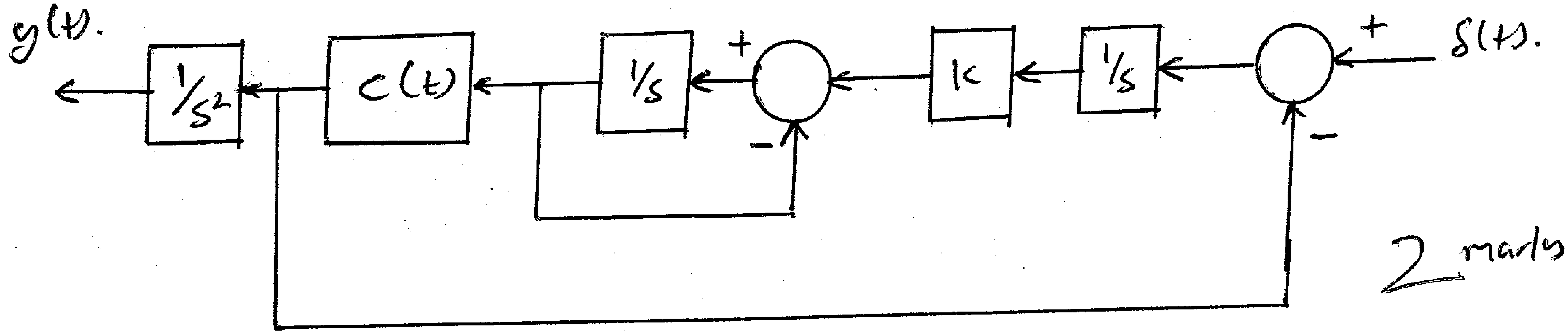


1 mark

Next, draw the adjoint block diagram by

1) ~~into~~ replacing summers by junctions & vice versa
2) reversing arrows,
3) replacing t by $t_F - t$.

This yields the following adjoint system.



2 marks.