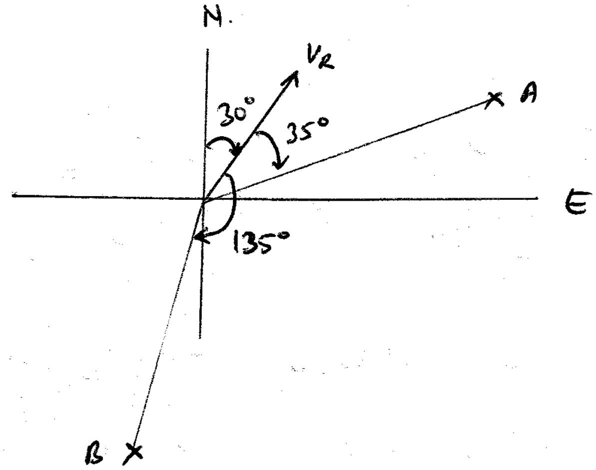
Tutorial 2. Aug. 16, 2007.

1).



Velocity of craft relative to water is $V_R = \begin{bmatrix} 5\sin 30 \\ 5\cos 30 \end{bmatrix}$.

Let drift velocity of water $V_D = \begin{bmatrix} V\sin\theta \\ V\cos\theta \end{bmatrix}$, θ - course of the drift.

The range rate measurements result from the total velocity $V_R + V_D$ of the craft relative to shore.

The range rate measurements are related to the total velocity by

$$\dot{\rho}_A = \begin{bmatrix} \sin\theta_A & \cos\theta_A \end{bmatrix} (V_R + V_D)$$

& $$\dot{\rho}_B = \begin{bmatrix} \sin\theta_B & \cos\theta_B \end{bmatrix} (V_R + V_D)$$

where $\theta_A = 35 + 30°$ is the true bearing to A

& $\theta_B = 135 + 30 = 165°$ is the true bearing to B.

$$\therefore \begin{bmatrix} \sin 65° & \cos 65° \\ \sin 165° & \cos 165° \end{bmatrix} \begin{bmatrix} 5\sin 30° + V\sin\theta \\ 5\cos 30° + V\cos\theta \end{bmatrix} = \begin{bmatrix} 3.3 \\ -2.5 \end{bmatrix}$$

$$\therefore V\begin{bmatrix}\sin\theta \\ \cos\theta\end{bmatrix} = \frac{1}{\sin(65-165^\circ)}\begin{bmatrix}\cos 165 & -\cos 65 \\ -\sin 165 & \sin 65\end{bmatrix}\begin{bmatrix}3.3 \\ -2.5\end{bmatrix} - \begin{bmatrix}5/2 \\ 5\sqrt{3}/2\end{bmatrix}$$

$$= \begin{bmatrix}2.163 \\ 3.168\end{bmatrix} - \begin{bmatrix}2.5 \\ 4.33\end{bmatrix} = \begin{bmatrix}-0.337 \\ -1.162\end{bmatrix}$$

$\therefore V = 1.21$ knots $\rightarrow$ magnitude of drift

$\theta = 196.17^\circ$. $\rightarrow$ true course angle of drift

3. On Aug. 16, 2007, (DATA FROM www.tecepe.com.br)

GHA of Aries = $249^\circ 47.5' = 249.8^\circ$

SHA of Vega = $80^\circ 41.7'$

$\therefore$ GHA of Vega $t_1 =$ SHA + GHA $= 330.486^\circ$

Declination of Vega $\delta_1 = 38^\circ 47.5' = 38.79^\circ$

$\therefore$ Unit vector to Vega is

$$u_1 = [\cos\delta\cos t,\ -\cos\delta_1 \sin t_1,\ \sin\delta_1]^T$$

$$= [0.6783 \quad 0.384 \quad 0.6264]^T$$

SHA of Deneb $= 49.57^\circ$

$\therefore$ GHA of Deneb $t_2 = 299.361^\circ$

Declination $\delta_2 = 45^\circ 18.5' = 45.308^\circ$

$\therefore$ unit vector to Deneb is

$$u_2 = [\cos\delta_2\cos t_2 \quad -\cos\delta_2 \cos t_2 \quad \sin\delta_2]^T$$

$$= [0.3448 \quad 0.613 \quad 0.711]^T$$

The zenith angles are

Vega: ~~90~~ $z_1 = 90 - 47.65° = 42.35°$

Deneb: $z_2 = 90 - 61.43° = 28.57°$

$$g = u_1 \times u_2 = \begin{bmatrix} 0 & -0.6264 & 0.384 \\ 0.6264 & 0 & -0.6783 \\ -0.384 & 0.6783 & 0 \end{bmatrix} \begin{bmatrix} 0.3448 \\ 0.613 \\ 0.711 \end{bmatrix}$$

$$= \begin{bmatrix} -0.11 & -0.2663 & 0.2833 \end{bmatrix}^T$$

$$h = \cos z_2 u_1 - \cos z_1 u_2$$

$$= \cos(28.57) \begin{bmatrix} 0.6783 \\ 0.384 \\ 0.6264 \end{bmatrix} - \cos(42.35°) \begin{bmatrix} 0.3448 \\ 0.613 \\ 0.711 \end{bmatrix} = \begin{bmatrix} 0.3408 \\ -0.1158 \\ 0.0246 \end{bmatrix}$$

$$(g \times h) = \begin{bmatrix} 0 & -0.2833 & -0.2663 \\ 0.2833 & 0 & 0.11 \\ 0.2663 & -0.11 & 0 \end{bmatrix} \begin{bmatrix} 0.3408 \\ -0.1158 \\ 0.0246 \end{bmatrix} = \begin{bmatrix} 0.0263 \\ 0.099 \\ 0.1035 \end{bmatrix}.$$

$$a_2 = \frac{1}{g^T g} = 6.127 \qquad a_1 = \cancel{\pm} \frac{\sqrt{g^T g - h^T h}}{g^T g} = \cancel{\pm} 1.098$$

The two points at the intersections of the two LOPs are

$$r_1 = a_1 g + a_2 (g \times h) = 1.098 \begin{bmatrix} -0.11 \\ -0.2663 \\ 0.2833 \end{bmatrix} + 6.127 \begin{bmatrix} 0.0263 \\ 0.099 \\ 0.1035 \end{bmatrix} = \begin{bmatrix} 0.040 \\ 0.3141 \\ 0.9452 \end{bmatrix}$$

$\therefore$ ~~latitude~~ longitude is given by $\tan \lambda_1 = \frac{0.3141}{0.04} = 7.8525 \quad \therefore \lambda_1 = 82.74°$.

~~longitude~~ latitude $\sin\varphi_1 = 0.9452 \Rightarrow \varphi_1 = 70.94^\circ$.

The other point is

$$r_2 = -a_1 g + a_2 (g \times n)$$

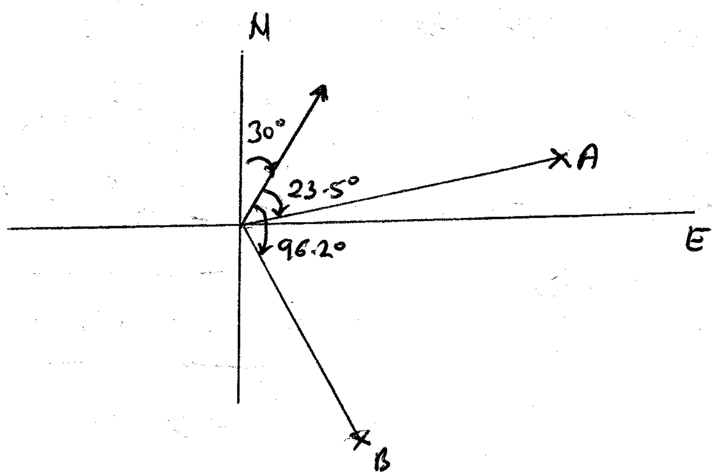
$$= -1.098 \begin{bmatrix} -0.11 \\ -0.2663 \\ 0.2833 \end{bmatrix} + 6.127 \begin{bmatrix} 0.0263 \\ 0.099 \\ 0.1035 \end{bmatrix} = \begin{bmatrix} 0.2819 \\ 0.8982 \\ 0.3230 \end{bmatrix}$$

longitude, ∴ $\tan\lambda_2 = \dfrac{0.8982}{0.2819} = 3.186 \Rightarrow \lambda_2 = 72.57^\circ$

~~longitude~~ latitude $\sin\varphi_2 = 0.3230$ ∴ $\varphi_2 = 18.84^\circ$.

The correct ~~answer is~~ coordinates for Mumbai are 18.84°N, 72.57°E, since Mumbai is to the south of the tropic of Cancer.

2



Choosing the origin at A, $r_A = [0 \;\; 0]^T$, $r_B = [-0.8, -0.35]^T$.

At the 1st observation, ~~θ_A~~ the bearings of A & B are

$$\theta_{A_1} = 30 + 23.5^\circ = 53.5^\circ$$

$$\theta_{B_1} = 30 + 96.2^\circ = 126.2^\circ$$

$\therefore$ Position fix from the 1st observation is

$$\begin{bmatrix} \cos\theta_{A_1} & -\sin\theta_{A_1} \\ \cos\theta_{B_1} & -\sin\theta_{B_1} \end{bmatrix} r_1 = \begin{bmatrix} 0 \\ -0.8\cos\theta_{B_1} + 0.35\sin\theta_{B_1} \end{bmatrix}$$

$$\therefore r_1 = \frac{1}{\sin(\theta_{A_1} - \theta_{B_1})} \begin{bmatrix} -\sin 126.2^\circ & \sin 53.5^\circ \\ -\cos 126.2^\circ & \cos 53.5^\circ \end{bmatrix} \begin{bmatrix} 0 \\ 0.75 \end{bmatrix}$$

$$\therefore r_1 = \begin{bmatrix} -0.6356 & -0.4703 \end{bmatrix}^T$$

for the 2nd observation, $\theta_{A_2} = 65^\circ$, $\theta_{B_2} = 137.5^\circ$

$$\begin{bmatrix} \cos\theta_{A_2} & -\sin\theta_{A_2} \\ \cos\theta_{B_2} & -\sin\theta_{B_2} \end{bmatrix} r_2 = \begin{bmatrix} 0 \\ -0.8\cos\theta_{B_2} + 0.35\sin\theta_{B_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8262 \end{bmatrix}$$

$$\therefore r_2 = \frac{1}{\sin(65 - 137.5)} \begin{bmatrix} -\sin 137.5^\circ & \sin 65^\circ \\ -\cos 137.5^\circ & \cos 65^\circ \end{bmatrix} \begin{bmatrix} 0 \\ 0.8262 \end{bmatrix} = \begin{bmatrix} -0.785 \\ -0.3661 \end{bmatrix}$$

$\therefore$ True velocity $V_T = \frac{1}{t}(r_2 - r_1) = \begin{bmatrix} -1.793 \\ 1.25 \end{bmatrix}$ knots.

$V_T = V_R + D_D$.

Relative velocity $V_R = \begin{bmatrix} 1.4\sin 320^\circ & 1.4\cos 320^\circ \end{bmatrix}^T = \begin{bmatrix} -0.899 & 1.072 \end{bmatrix}^T$

$\therefore$ drift velocity $V_D = V_T - V_R = \begin{bmatrix} -0.894 & 0.178 \end{bmatrix}$.

$\therefore$ drift $= 0.911$ knots with a true course of 281.26°.