AE 457/641 – Navigation and Guidance

Tutorial 3, September 6, 2007

1. A zero-mean 2D random vector x has the covariance matrix P_{xx}. Let λ_1 and λ_2 be the eigenvalues of P_{xx}, and let v_1, v_2 be the corresponding eigenvectors. Assume that $\lambda_1 \neq \lambda_2$, and that v_1 and v_2 are normalized to have unit magnitude. Let V be the matrix having row vectors $\lambda_1^{-\frac{1}{2}} v_1^{\mathrm{T}}$ and $\lambda_2^{-\frac{1}{2}} v_2^{\mathrm{T}}$. Define $y = Vx$. Find the mean and covariance matrix of y. If x is Gaussian and

$$P_{xx} = \begin{bmatrix} 2.5 & -0.5 \\ -0.5 & 2.5 \end{bmatrix},$$

then write y in terms of x, and find the radius of the disc in which y takes values with probability 0.9. (Hint: The eigenvectors of a symmetric matrix having distinct eigenvalues are orthogonal.) (**Mayur Singh** (04D01003) + teammate)

2. Using range measurements to two points having coordinates (0,0) and (0,3.5), an observer deduces his coordinates to be (1.2,1.3). If the range measurement error vector is zero-mean and has the covariance matrix

$$\begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.15 \end{bmatrix},$$

then find the covariance matrix of the position estimate error and the RMS value of the estimate error. Repeat the calculations in the case where the observer makes measurements that yield the position estimates (1.74, 1.75) and (0.8,0.1). Can you explain the difference in the RMS error values resulting from the three sets of observations? (**Tushar Malgonkar** (04011039) + teammate)