AE 457/641 – Navigation and Guidance

Tutorial 4, September 7, 2007

1. This problem illustrates the use of BLUE to combine position estimates, and requires you to know how to track uncertainty as it propagates all the way from measurements to position estimates. An aircraft uses LORAN-C stations P, Q, R, S and T located at coordinates $(0, 0)$, $(0, 1034)$, $(1156, 0)$, $(2022, 1179)$, and $(1126, 2163)$, respectively, to estimate its position. Using the differences in ranges to stations P-Q and Q-R, the position of the aircraft is estimated to be $(1000.67, 878.39)$. Simultaneously, differences in ranges to stations R-S and R-T yield $(1001.07, 877.76)$ as an estimate of the aircraft position. If the errors in the synchronization of the transmitters are zero-mean, mutually uncorrelated, and have standard deviations $\sigma_{\mathrm{P}} = 1$ nanosecond, $\sigma_{\mathrm{Q}} = 1.5$ nanoseconds, $\sigma_{\mathrm{R}} = 2.5$ nanoseconds, $\sigma_{\mathrm{S}} = 1.5$ nanoseconds, and $\sigma_{\mathrm{T}} = 2$ nanoseconds, then

   (a) Find the variances of and variances between the errors incurred in measuring the range differences l_{PQ}, l_{QR}, l_{RS} and l_{RT}.

   (b) Find the combined covariance matrix for the errors in the two position estimates, keeping in mind the possibility that the errors in the two estimates may not be uncorrelated.

   (c) Find the best linear unbiased estimate of the aircraft position. Compare RMS error values for the individual estimates as well as the best estimate.

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