AE 457/641 – Navigation and Guidance

Tutorial 8, November 14, 2007

1. A cannonball launched from $(0,0)$ has to hit a target at $(R,0)$. Write down the hit equation for the Cartesian components u_0 and v_0 of the initial velocity vector of the cannonball. If the perturbation in the initial velocity vector has the covariance matrix

$$\begin{bmatrix} \sigma_{uu}^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_{vv}^2 \end{bmatrix},$$

then find the variance in the miss distance. What initial velocity vector leads to the smallest variance for the terminal miss distance? (Assume a flat earth and uniform gravity.) (**Ranjan Kumar** (02D01007))

2. This problem concerns Lambert's problem and Lambert guidance. (**Vishal Prabhu** (04D11019))

   (a) Write a Matlab program to solve Lambert's problem for a ballistic missile flying in a central inverse square gravitational field without atmospheric drag. Use the program to find the solution when $r(0) = r(t_{\mathrm{f}}) = R_{\mathrm{e}}$, $\phi(0) = 0$, $\phi(t_{\mathrm{f}}) = 45°$, and $t_{\mathrm{F}} = 30$ minutes. Plot the resulting trajectory.

   (b) Simulate and plot the trajectory of the missile under Lambert guidance. Assume $r(0) = r(t_{\mathrm{f}}) = R_{\mathrm{e}}$, $\phi(0) = 0$, $\phi(t_{\mathrm{f}}) = 45°$, and $t_{\mathrm{F}} = 30$minutes from launch. Assume the thrust acceleration of the missile to be 20 m/s^2. Assume the same initial speed as calculated in part a) above, with a launch heading error of 5° from that obtained in part a). On the same figure, also plot the trajectory that results if no guidance is used.

3. This problem concerns the effect of atmospheric drag on the trajectory of a ballistic missile. Assume a ballistic missile is launched from the surface of the earth at a flight path angle of 45° with a speed of 1km/s. The missile has a drag acceleration per unit dynamic pressure of 2.04×10^{-4} m^2/kg. Assume that the density ρ (in kg/m^3) varies with altitude h (in meters) as follows:

$$\begin{aligned} \rho &= 1.225e^{-h/9144},\ h \leq 9144, \\ &= 1.752e^{-h/6705},\ h > 9144. \end{aligned}$$

Write down equations of motion of the missile assuming the earth to be a perfect homogeneous sphere. Plot the trajectories of the missile till intercept both with and without drag. Find the time of flight, range and maximum altitude for both trajectories. (**Nabjit Barman** (07301409) and **Karthikeyan** (07301803))

4. Use the variational equation associated with the dynamics of the ballistic missile described in the problem above to (numerically) compute sensitivity of the location of the intercept point to small perturbations in the launch altitude, all other initial conditions remaining the same. (**Chinmay Rajhans** (07301018) and **Arun Abraham** (06301012))

Common data: μ for earth $= 3.98601 \times 10^5$ km^3/s^2, $R_{\mathrm{e}} = 6378.135$km.