AE 695 – State Space Methods

Quiz 2, Thursday, 28/09/06, 4:15pm-5pm, Closed Notes, 10 marks

1. Give an example of a matrix $A \in \mathbb{R}^{4\times 4}$ such that $A^k = 0$ for every $k = 3, 4, \ldots,$ but $A^2 \neq 0$. (2)
2. Let x, y be nonzero vectors in $\mathbb{R}^n$, and let $A = xy^{\mathrm{T}}$. Find the range and kernel of A. For what $z \in \mathbb{R}^n$ does the equation $Ap = z$ have a solution $p \in \mathbb{R}^n$? (3)
3. Suppose $A \in \mathbb{R}^{n\times m}$ and $B \in \mathbb{R}^{n\times l}$. Show that $\operatorname{range}[A\ B] = \operatorname{range} A + \operatorname{range} B$. (3)
4. $A \in \mathbb{R}^{15\times 15}$ has an eigenvalue $\lambda \in \mathbb{C}$ with algebraic multiplicity 10. The ranks of $(\lambda I - A)^k$ for various values of k are shown below. Write down the Jordan block of A associated with λ. (2)

k	1	2	3	4	5
rank	12	9	6	5	5

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