AE 695 – State Space Methods

Quiz 3, Thursday, 19/10/06, 3:30pm-5pm, Closed Notes, 15 marks

1. Use series expansion to compute e^{At} for $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. (2)

2. Use Laplace transforms to compute e^{At} for (3)

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Verify that $X(t) = e^{At}Ce^{Bt}$ is the solution of the matrix differential equation $\dot{X} = AX + XB$ satisfying $X(0) = C$. (2)

4. Suppose the matrix exponential of the partitioned matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$ is itself partitioned (consistent with the partitioning of A) as $e^{At} = \begin{bmatrix} B_{11}(t) & B_{12}(t) \\ B_{21}(t) & B_{22}(t) \end{bmatrix}$. Show that $B_{21}(t) = 0$ for every t, while $B_{ii}(t) = e^{A_{ii}t}$ for $i = 1, 2$. (3)

5. Suppose $A \in \mathbb{R}^{n\times n}$ is asymptotically stable and $Q \in \mathbb{R}^{n\times n}$ is symmetric and positive definite. Let $P = \int_0^\infty e^{A^{\mathrm{T}}t}Qe^{At}dt$. Show that P is positive definite and satisfies $A^{\mathrm{T}}P + PA = -Q$. (3)

6. Suppose $A \in \mathbb{R}^{n\times n}$ satisfies $A^{\mathrm{T}}P + PA = 0$ for some symmetric positive-definite matrix P. Show that every eigenvalue of A is imaginary. (2)

7. **(Optional Bonus Question)** Show that the matrix A in the question above is Lyapunov stable, that is, every eigenvalue of A is also semisimple. (3)