AE 695 – State Space Methods

Quiz 3, Thursday, 15/11/07, 3:45pm-5pm, Open Notes, 15 marks

ONLY ONE'S OWN HANDWRITTEN NOTES ARE PERMITTED. PHOTOCOPIES, PRINTED MATTER ARE NOT ALLOWED.

1. Show that the controllability Grammian $P(t) = \int_0^t e^{A\tau} B B^{\mathrm{T}} e^{A^{\mathrm{T}}\tau} d\tau$ satisfies the matrix differential equation $\dot{P}(t) = AP(t) + P(t)A^{\mathrm{T}} + BB^{\mathrm{T}}$ with initial condition $P(0) = 0$. (3)

2. Suppose the symmetric, positive-definite matrix $P \in \mathbb{R}^{n\times n}$ satisfies $A^{\mathrm{T}}P + PA + \mu P = -Q$, where $A \in \mathbb{R}^{n\times n}$, $\mu \in \mathbb{R}$ is positive, and $Q \in \mathbb{R}^{n\times n}$ is a symmetric positive-definite matrix. Show that every eigenvalue of A has real part less than $-\mu$. (3)

3. Consider the system $\dot{x} = Ax + Bu$ and $y = Cx$, where
$$A = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 3 & 6 \\ -5 & -1 & -4 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 2 \end{bmatrix}.$$
Does there exist an input which steers the system from the initial state $x_{\mathrm{i}} = 0$ to the state $x_{\mathrm{f}} = [0\ 1\ 1]^{\mathrm{T}}$? Are the outputs generated by initial conditions $x_1 = [3\ 2\ 0]^{\mathrm{T}}$ and $x_2 = [4\ 3\ -1]^{\mathrm{T}}$ in response to a given input same or different? Explain. (5)

4. In the problem above, verify that 2 and -3 are eigenvalues of A. Is the eigenvalue 2 controllable? Is the eigenvalue -3 observable? (4)

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