Sample Question Paper for PhD Admission

Part-I

The Life of the Mind (extracted from Our Oriental Heritage – Will Durant)

As in our middle ages, the scientists of India, for better and for worse, were priests.

Astronomy was an incidental offspring of astrology, and slowly emancipated itself under Greek influence. The earliest astronomical treatises, the Siddhantas (ca. 425 B.C.), were based on Greek science, and Varahamihira, whose compendium was significantly entitled Complete System of Natural Astrology, frankly acknowledged his dependence upon the Greeks. The greatest of Hindu astronomers and mathematicians, Aryabhata, discussed in verse such poetic subjects as quadratic equations, sines and the value of PI; he explained eclipses, solstices and equinoxes, announced the sphericity of the earth and its diurnal revolution on its axis, and wrote, in daring anticipation of Rennaissance science: “The sphere of the stars is stationary, and the earth, by its revolution, produces the daily rising and setting of planets and stars.” His most famous successor, Brahmagupta, systematized the astronomic knowledge of India, but obstructed its development by rejecting Aryabhata’s theory of the revolution of the earth. These men and their followers adapted to Hindu usage the Babylonian division of the skies into zodiacal constellations; they made a calendar of twelve months, each of thirty days, each of thirty hours, inserting an intercalary month every five years; they calculated with remarkable accuracy the diameter of the moon, the eclipses of the moon and the sun, the position of the poles, and the position and motion of the major stars. They expounded the theory, though not the law, of gravity when they wrote in the Siddhantas: “The earth, owing to its force of gravity, draws all things to itself.”

Please read the above paragraph and answer the following questions:

1. As per the author, who were the scientists of Ancient India?
2. What were the earliest astronomical treatises of India called?
3. What did Varahamihira compile and what did he frankly acknowledge?
4. According to the author, who was the greatest of Hindu astronomers and mathematicians?
5. How many hours were there in a day as per the calendar developed by Aryabhata and his followers?

General Engineering and Mathematics

1. Find the eigenvalues of the matrix A^{-1}, where $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

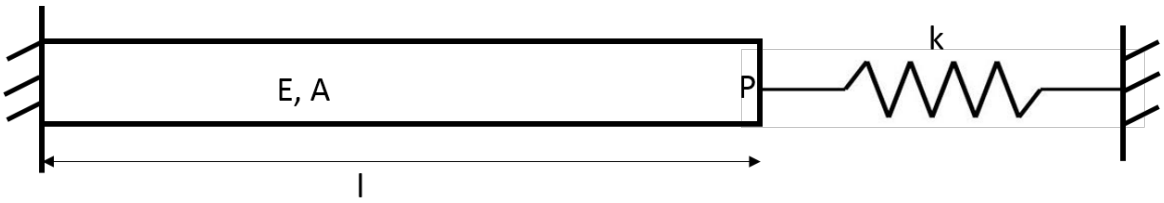
2. Find the maxima of the function $x^3 - 9x^2 + 24x + 39$.
3. The shadow of a building increases by 20 meters and the angle of elevation of the sunray decreases from 60^o to 45^o. What the height of the building?

4. A boat travels against the stream of a river from A to B in 8 hours. The same boat returns from B to A in the same direction as the stream in 7 hours. Find the ratio of the speed of the boat (in still water) to the speed of the river stream.

5. $\lim_{x \to 0} \frac{\sin x}{xe^x} = ?$

Section-A: STRUCTURES SPECIALIZATION

1. Consider a bar having Young's modulus E, cross-sectional area A and length *l*, as shown in the figure below. The end of the bar at the point P is connected to a spring whose stiffness is k. Determine the displacement of the bar due to a force F at the point P.



2. For a cantilever beam the two dimensional small strain theory gives the following strains:

$$\varepsilon_{xx} = A\,x\,y;\ \varepsilon_{yy} = -vA\,x\,y;\ 2\varepsilon_{xy} = A\,(1+v)(a^2 - y^2)$$

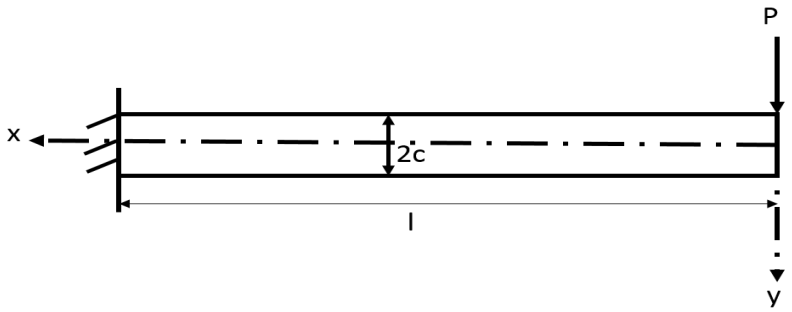
Determine if the strain fields satisfy the compatibility equation or not.

3. Suppose that a cantilever having a narrow rectangular cross-section of unit width, length *l* and height $2c$ is bent by a force P applied at its end. The upper and lower edges of the cantilever are free from load, and shearing forces, having a resultant P, are distributed

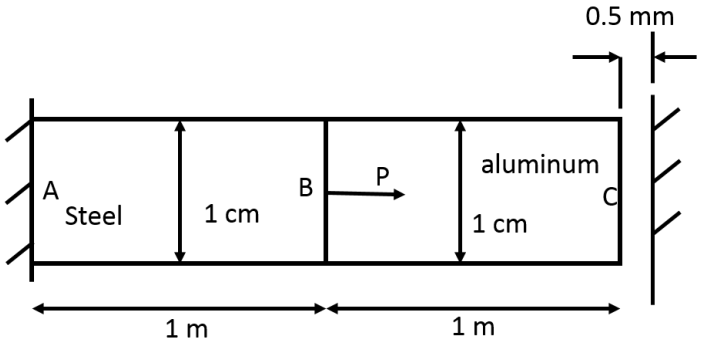
along the end $x = 0$. The end $x = l$ is fixed. Suppose that the stress function

$$\emptyset = Axy + \left(\frac{B}{6}\right) xy^3$$

satisfies the equilibrium equation for this problem. Determine the coefficients A and B such that the stress function also satisfies the boundary conditions of the problem.



4. A cylinder is attached to a wall at point A by a fixed support as shown in the diagram below. The half of the cylinder from point A to point B is made of steel ($E = 200\ GPa$) and has a length of $1\ m$ and a diameter of $1\ cm$; the half of the cylinder from point B to point C is made of aluminum ($E = 70\ GPa$) and has a length of $1\ m$ and a diameter of $1\ cm$. Before the cylinder is loaded, the free end (point C) is separated from a second wall by a distance of $0.5\ mm$.



At what value of the load P does the cylinder first touch the second wall?

5. For the problem described in Question 4, at what value of the load P does the strain at the points between B and C equal 0.1%?.

6. The state of stress at a particular point relative to the xyz coordinate system is given by the stress matrix.

$$\begin{bmatrix} 15 & 10 & -10 \\ 10 & 10 & 0 \\ -10 & 0 & 40 \end{bmatrix} MPa$$

Determine the normal stress on a surface intersecting the point and parallel to the plane

given by the equation

$$2x - y + 3z = 9$$

7. The following data are given for a vibrating system with viscous damping: $m = 10kg$, $k = 30\ N/m$, and $c = 0.12\ N/m/s$. Determine the ratio of any two successive amplitudes.

8. Consider a pin-ended column of modulus E, cross-sectional area A, Bending moment of Inertia I, and length L. This column is subjected to a rise of temperature from ΔT, and undergoes buckling at an Euler buckling load of P_{cr}. Develop an expression for coefficient of thermal expansion, α for the material in terms of the buckling load and the material and geometric properties of the column?
9. As a beam gets shorter in span and thicker in height, which assumption of simple beam theory becomes untenable?
10. What is the advantage of applying transverse loads at the shear center of a beam?

Section-B: DYNAMICS AND CONTROL SPECIALIZATION

1. For the characteristic equation given below, determine the value of the parameter $K \geq 0$ for which all the poles (1 real, 2 complex conjugate) will have -1 as the real part.

$$s^3 + 3s^2 + (K + 4)s + 4 + 0$$

2. Consider the following open loop system transfer function:

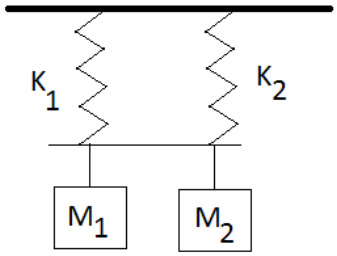
$$G(s) = \frac{K}{s(s^2 + 8s + 32)}; \quad K \geq 0; \quad s = \sigma + j\omega$$

Determine the point of crossing the imaginary axis of the root locus branches and the value of 'ω' for which it happens

3. Obtain frequency response of the following transfer function in terms of its magnitude & phase and determine the value of ω for which magnitude is 1.

$$G(s) = \frac{10(0.1s + 1)}{(s + 1)}; \quad s = \sigma + j\omega$$

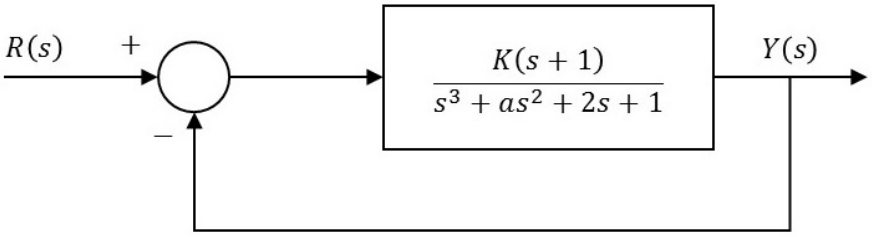
4. In the figure below, spring constant $K_1 = 4\ N/m$ and $K_2 = 12\ N/m$ and mass $M_1 = 3\ kg$ and $M_2 = 1\ kg$. What is the natural frequency in rad/sec?



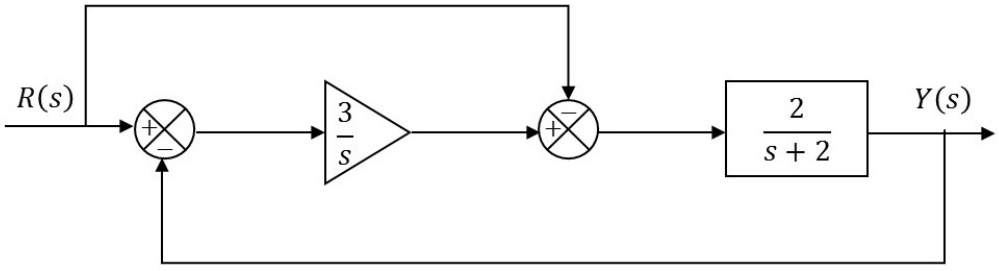
5. Given a unity feedback control system with $G(s) = \frac{K}{s(s+4)}$. Find the value of K for a damping ratio of 0.5.

6. Find the solution of the differential equation $\frac{dy}{dx} - y^2 = 0,\ (0 \le x \le 1)$, with $y(0) = 1$.

7. Find the values of K and a, when the feedback system shown below oscillates at 2 rad/sec.



8. Consider the closed loop control system shown below which is subjected to a unit step input. Find the steady state error value.



9. The unit impulse response of a second order under-damped system starting from rest is given by $c(t) = 12.5e^{-6t}\sin 8t,\ t \ge 0$. Find the steady-state value of the unit step response of the system.

10. Consider the state space representation of the following system:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

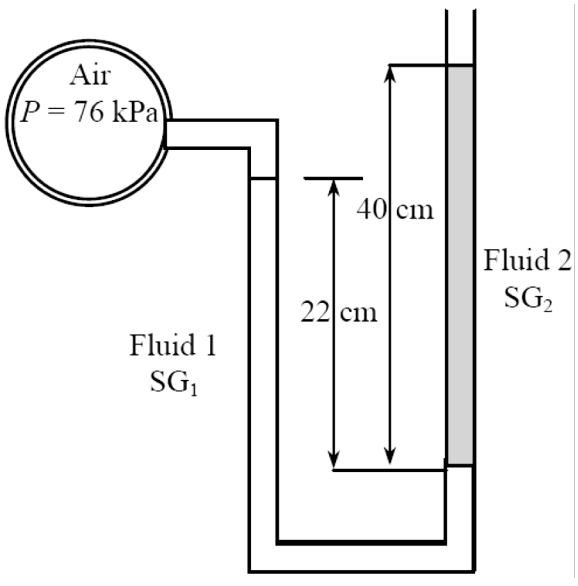
$$\boldsymbol{y}(t) = \begin{bmatrix} 0 & 1 & 3 \end{bmatrix} \boldsymbol{x}(t)$$

where $\boldsymbol{x}$, $\boldsymbol{u}$ and $\boldsymbol{y}$ are the state, input and output, respectively. Using the observability matrix, check the complete observability of this system.

Section-C: AERODYNAMICS AND PROPULSION SPECIALIZATIONS

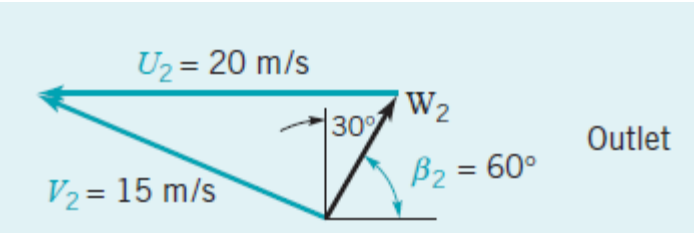
1. If the nozzle upstream pressure is $10^6 Pa$, what will be the pressure at the choked throat for a gas having $\gamma = 1.4$?

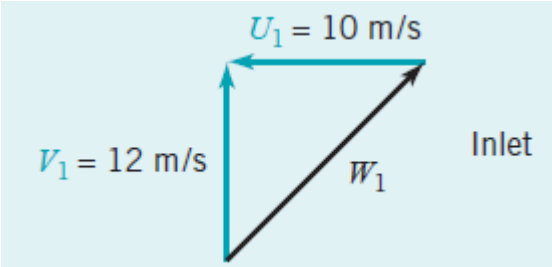
2. Consider a double-fluid manometer attached to an air pipe shown in Figure. If the specific gravity of fluid 1 is 13.55, determine the specific gravity of the other fluid for the indicated absolute pressure of air. Take the atmospheric pressure to be $100\ kPa$.



3. If 20 kJ are added to a Carnot cycle at a temperature of 100°C and 14.6 kJ are rejected at 0°C, determine the location of absolute zero on the Celsius scale.

4. Following are inlet and outlet velocity triangles where U, V and W represent blade speed, flow velocity and relative velocity respectively. Can you identify the class of turbomachine?



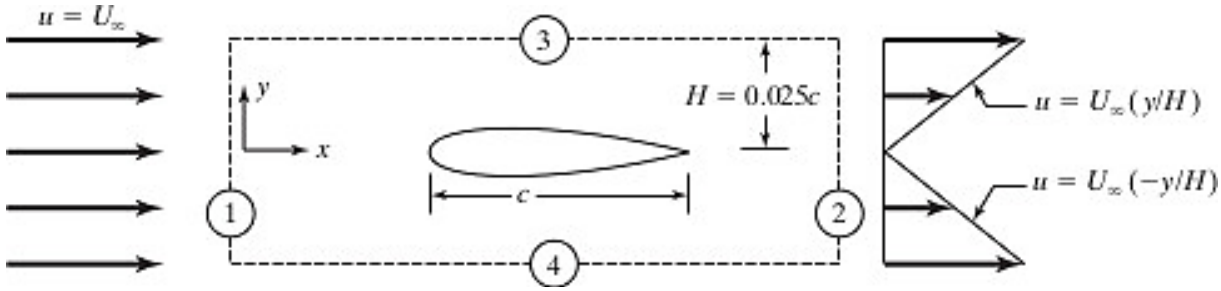


5. If In an ideal Brayton cycle based gas turbine engine, the air at the inlet is at 300K and 0.1MPa. The static pressure ratio of compressor is 6.25 and the maximum temperature is 1073K. Taking $\gamma = 1.4$ and $C_p = 1005$ J/kg.K for both compression and expansion process, find the cycle efficiency by calculating the net work done and the heat supplied per unit kg of air.

6. The flow in the narrow gap (of width h) between two concentric cylinders of length L, with the inner one of radius R rotating at angular speed ω, can be approximated by the Couette solution to the Navier-Stokes equations. Compute the torque T and power P required to rotate the shaft at a rotational speed of ω rad/s.

7. A flow-field on the x-y plane has the velocity components $u = 3x + y$ and $v = 2x - 3y$. Find the circulation $\Gamma = \int V.ds$ around the circle $(x-1)^2 + (y-6)^2 = 4$.

8. Velocity profiles are measured at the upstream end (surface 1) and at the downstream end (surface 2) of a rectangular control volume that contains an airfoil of chord c, as shown in the figure. If the flow is incompressible, two dimensional, and steady, what is the drag coefficient C_d for the airfoil? The vertical dimension H is *0.025c* and $C_d = \frac{d}{\frac{1}{2}\rho U_\infty^2 c}$ where d is the drag per unit span.



9. The fluid dynamic characteristics of a glider flying at 80 m/s near sea level are to be investigated with the aid of a 1:5 scale model. The model tests are to be performed in a wind tunnel using standard air at 27°C. Will such a test reveal useful results?

10. The square, flat plate shown in the figure (a) below is cut into four equal-sized pieces and arranged as shown in figure (b). Determine the ratio of the drag on the original plate [case (a)] to the drag on the plates in the configuration shown in (b). Assume laminar boundary flow.