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Closed Form Solution for Acoustic Localization of Ballistic Source

Sourav Bhattacharya
Master's Student
Dept. of Aerospace Engineering,
IIT Bombay
Email: srvb1995@gmail.com

Aniruddha Sinha
Assistant Professor
Dept. of Aerospace Engineering,
IIT Bombay
Email: as@aero.iitb.ac.in

Hemendra Arya
Associate Professor
Dept. of Aerospace Engineering,
IIT Bombay
Email: arya@aero.iitb.ac.in

Abstract
Ballistic sources produce a signature acoustic wave as muzzle blast in case of a gun which can be used to locate the source. The objective is to help security forces locate enemies in tough combat operations. This paper introduces an application of an existing algorithm to this problem by recording the acoustic signal of the event at multiple microphones. The equation involves the distance between the source and the sensors and the differential time-of-arrival (DTOA) of noise signals between a pair of sensors. Four microphones arrayed in the form of a tetrahedron are used as sensors. Firecrackers are burst at various locations around the sensor array. The results indicate that the estimated bearing angles are in very good agreement with their corresponding true values. The estimated ranges show large deviations from the true values. However, bearing angle is seen as a more important parameter than the range for our practical application.

Keywords: *differential time-of-arrival; Minkowski functional; omni-directional; sampling frequency; spherical coordinates; ground reflection; residual.*

I. INTRODUCTION

The motivation behind the localization of an acoustic source comes from keeping in mind of the situations modern day security forces deal when indulging in gun battle with various gangs in close and dense ranges. With modern combat equipments and enemy strategies, sometimes it is difficult for forces to locate gunfires easily and can be caught off guard. Knowing the real-time location of the attacker will help in more prompt and strategic action.

Bancroft's method [1] is an algorithm suggested for triangulation of GPS coordinates of a sensor. This paper deals with the converse problem of localization of gunshots; i.e., locating a source(gun) using a set of acoustic signals of the event obtained experimentally at multiple sensors. The Bancroft method gives a closed form solution for GPS equations. The algorithm discussed in this paper uses the differential time of arrival (DTOA) information of the sound event between pairs of sensors to calculate the location of the source.

The study is done for helping security men locate enemies with guns. So, it is more apt to obtain the results in spherical coordinates than in Cartesian coordinates. A human reacts and responds better to instructions in terms of angles and ranges rather than just coordinates of points in space. So the parameters chosen for study are bearing angle and range which are parameters of spherical coordinate system. The zenith angle which is another parameter for the 3-dimensional location in spherical coordinates; however it has been neglected for this introductory study. The results show that the algorithm gives a very good estimation of the bearing angle of the source. The estimated range, on the other hand, shows significant discrepancy from the true range when the algorithm is applied on the recorded data set.

II. METHODOLOGY

A. Experimental details

Figure 1: Microphone rig used in the experiment

The setup for the experiment consists of an arrangement of four dynamic microphone sensors making the vertices of a tetrahedron for recording the acoustic wave. The minimum number of microphones required for locating a point in 3-dimensional space is four. Placing the microphones in a tetrahedron arrangement maximizes the distance between any two microphones for a 3-dimensional geometrical arrangement with four vertices. The differential time-of-arrival (DTOA) depends directly on the inter-sensor distance. Thus, for a fixed sampling frequency, the best resolution of DTOA is possible with a tetrahedron. The microphone being used in the experiment is Sennheiser MD 42. It is an omni-directional microphone with a sensitivity of 2 mV/Pa and nominal impedance of 350 Ω. The microphone setup used in the experiment is shown in Figure 1.

An impulsive sound event is created by bursting a fire cracker. The acoustic wave from the source reaches the sensors at different instances depending on the location of the source and the corresponding sensors. A captured signal of an event is shown in Figure 2.

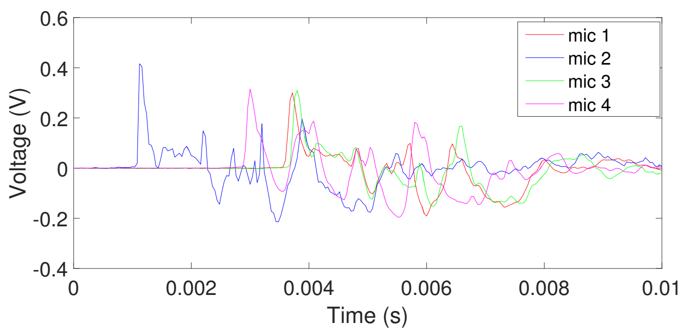

Figure 2: Captured signal of an event at all 4 microphones.

The sensor data is obtained using a Data AcQuisition (DAQ) board connected to the sensors for processing. The DAQ board used for the experiment is from Measurement Computing Corporation and the model used is USB-1608FS. It is a 16-bit, 8 channel, simultaneous sampling DAQ device. The sampling frequency of the acquisition board for the experiment was set at 25kHz for each sensor. The DTOA of the signal between the sensors are computed by applying the method of cross-correlation upon the voltage vs time data obtained using the DAQ board between sensor 1 (reference) with sensor 2,3 and 4 respectively and fed as an input to the algorithm.

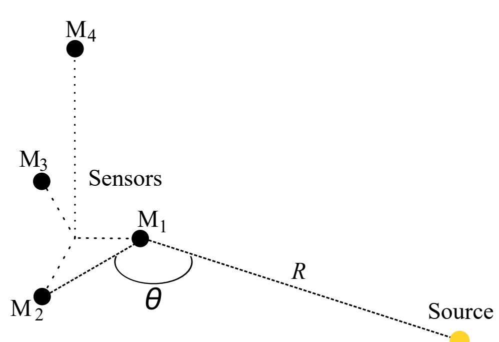

Figure 3: Schematic of sensor arrangement used in the experiment

The measurement of lengths are done using threads and measuring tape, due to which it will have some degree of human and mechanical error. A schematic of arrangement of the sensors is shown in Figure 3. The triangular array of sensors makes the x-y plane. The line joining sensor 1 and the centroid of the array is taken as the reference x-axis The range R and the bearing angle θ of a source from the M_1-M_2 axis is shown in Figure 3.

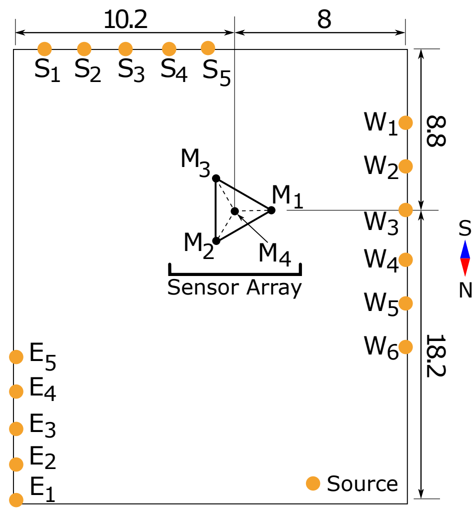

Figure 4: Top view of the experimental setup – all dimensions are in meters

Firecrackers were burst from 16 locations around the sensor array. The top view of the experimental setup is shown in Figure 4. The sensors are marked as [M_1,..,M_4]. The arrangement of sensors is such that M_1, M_2 and M_3 form a horizontal plane and M_4 is at a height vertically above the centroid of the triangle formed by M_1, M_2 and M_3. The rectangular area shown in the top view represents the actual rooftop where the experiment was performed.

B. Numerical details

The localization situation considered in the paper is the converse of the original situation which Bancroft suggested for calculating GPS location. Here, signal emitted from a source is recorded simultaneously at multiple sensors (4 in this case) to determine the location of the source using the DTOA data. Assuming propagation of sound with uniform speed 'c', we can write

$$ct_i = d(\boldsymbol{x}, \boldsymbol{m_i}) + c\tau \qquad \forall\, i \in [1, 2, ..., n] \tag{1}$$

where, $d(\boldsymbol{x}, \boldsymbol{m_i})$ is the Euclidean distance between the source with coordinates $\boldsymbol{x}$ and the ith sensor with coordinates $\boldsymbol{m_i}$; t_i is the time of arrival of the signal for the ith sensor; τ is the time at which the signal is generated at the source.

Taking sensor 1 as reference, we can subtract Eq. (1) for the reference sensor (i = 1) from Eq. (1) for ith sensor ($i \neq 1$) which gives

$$ct_i - ct_1 = d(\boldsymbol{x}, \boldsymbol{m_i}) + c\tau \; - d(\boldsymbol{x}, \boldsymbol{m_1}) - c\tau$$

which can be written as

$$ct_{i1} = d(\boldsymbol{x}, \boldsymbol{m_i}) + cb \qquad \forall i \in [2, 3, ..., n] \tag{2}$$

where, ct_{i1} is the DTOA of the signal at the ith sensor (w.r.t sensor 1) and the term b is equal to $-d(\boldsymbol{x}, \boldsymbol{m_1})/c$.

The Bancroft method gives a least-squares solution of an overdetermined system of equations [2]. The equation used in the method contains the distance term which involves square root. This is removed by a "squaring" process which leads to two solutions. The residual or the error term is calculated by taking the ct_{i1} in Eq. (2) to other side. The new RHS term is then squared and summed over the sensors. The error term is

$$\text{err} = \sum_{i=2}^{n} [d(\boldsymbol{x}, \boldsymbol{m_i}) + cb - ct_{i1}]^2 \tag{3}$$

where the notations are same as stated above in Eq. (2).

An important step before the localization algorithm is applied on the experimental dataset is a calibration process to get the calibrated sensor array coordinates. It is done to take care of the measurement bias and errors. The calibration algorithm takes an initial guess of the sensor coordinates as an input and implements a non linear least-squares method to get the calibrated sensor coordinates. The cost function is the sum of squares of error between propagation path differences from source to a sensor pair and the DTOA for the same pair multiplied by speed of sound. The sum is over all sensors (other than the common sensor in the pairings), and all events. The calibrated coordinates comes out very close to the measured coordinate values as will be discussed later. The calibrated sensor array coordinates were then used in the localization algorithm instead to get a better solution from the method.

III. RESULTS

A. Calibration results

Table 1: **Comparison of Measured location and Calibrated location of sensors – all in meters**

S.No	**Measured** (x, y, z)	**Calibrated** (x, y, z)
M_1	(1.096, 0, 0.675)	(1.082, -0.011, 0.679)
M_2	(0.187, -0.525, 0.675)	(0.184, -0.512, 0.672)
M_3	(0.187, 0.525, 0.675)	(0.193, 0.53, 0.671)
M_4	(0.49, 0, 1.595)	(0.499, 0.007, 1.605)

Two sets of data were recorded for the 6 sources [W_1,W_2..,W_6]. The first dataset was used for the calibration of the sensor locations and the second set for the validation of the algorithm. The measured sensor locations and the calibrated sensor locations are compared in Table 1. The results obtained from the calibration method demonstrate that the difference in the measured and calibrated sensor locations are very small. The origin of the axes is at a distance 49 cm from the centroid of the array along the negative x-axis.

B. Validation results

The second dataset for [W_1,W_2..,W_6] along with [S_1,S_2..,S_5] and [E_1,E_2..,E_5] dataset were used for validating the algorithm. The algorithm takes the calibrated sensor coordinates and their corresponding difference in the time of arrival of the sound wave w.r.t a reference sensor as input. The coordinates of the source is given as the output from which the bearing and the range of the source is computed as they are seen as more useful parameters.

Table 2: **Comparison of bearing angles for each source all – measured, estimated and the error thereof – all in degrees**

S.No	**Measured**	**Estimated**	**Error**
W_1	189.49	191.23	1.74
W_2	172.23	172.03	0.20
W_3	150.00	150.77	0.77
W_4	127.77	130.15	2.37
W_5	110.74	107.42	3.32
W_6	99.36	102.12	2.76
S_2	278.98	277.49	1.50
S_3	274.25	272.65	1.60
S_4	261.77	260.43	1.34
S_5	254.68	272.51	17.83
E_1	29.17	29.39	0.22
E_2	24.78	23.14	1.64
E_3	18.55	19.81	1.26
E_4	10.37	8.05	2.32

The measured and obtained bearings for all the sources but S_1 and E_5 and their corresponding error values (in degrees) are stated in Table 2. The bearing of the sources obtained from the solution of the algorithm and the measured bearing values show a difference of not more than 3.3 degrees with an exception for point S_5. Point S_5 shows a significant deviation of 17.83 degrees from the true value. Inspection of the raw microphone signals revealed that significant ground reflection effects were present, confounding the cross-correlation calculation. The captured signals show broad peaks for the ground reflection of the actual events. The cross-correlation tool picks the slightly delayed reflected peaks rather than the actual event for its time delay calculation resulting in such a large error. Same argument is true for the two points S_1 and E_5 which have not been shown in the bearing and range tables. In case of S_1 and E_5, this was so severe that the result was coming out to be a complex number. Efforts are being made to make the DTOA computation more robust. The study done applies a process of removing the ground reflection from the signal [3] but it has proved ineffective for the defective cases as the main event and reflection peaks have almost similar amplitude.

The comparison between the measured range values and the estimated values from the algorithm are shown in Table 3. The error in range is normalized w.r.t to measured range for each location which is also shown in Table 3. The range obtained from the algorithm shows a big deviation from the measured range. The error in range is very high because the range is more sensitive to the sampling frequency as compared to the bearing. The increase in sampling frequency have been found to improve the range estimation in further numerical simulations which are not part of this paper. The sampling frequency of the captured signal has proved to be a critical parameter in the analysis of the solutions obtained from the algorithm. A slight change in the DTOA value for the sensors results in big changes in the obtained solution. So, getting the sample closest to actual time of arrival of the event gives the best results which directly depends on the sampling frequency.

Table 3: **Comparison of range for each source – measured, estimated – both in metres and the error normalized w.r.t measured value**

S.No	Measured	Estimated	Normalized Error
W_1	9.67	6.30	0.35
W_2	8.06	13.13	0.63
W_3	7.46	12.46	0.67
W_4	8.06	24.63	2.05
W_5	9.64	3.39	0.65
W_6	11.77	298.73	24.38
S_2	11.42	190.72	15.70
S_3	10.74	63.23	4.89
S_4	9.56	29.28	2.06
S_5	9.18	1.38	0.85
E_1	21.18	19.66	0.07
E_2	18.83	10.79	0.43
E_3	16.40	28.91	0.76
E_4	14.25	6.63	0.53

The Bancroft method gives two solutions for source locations for a single set of inputs. The residual as discussed earlier is calculated for both the solutions. It serves as a criterion for validating an obtained solution. In some cases, for a particular source only one solution was found to be valid which was clearly evident from the error term. But, in some cases both the solutions for a particular source had very small error terms suggesting that both solutions satisfied Eq. (2) for the given input values. In such cases, it was found that one solution gave the source locations very close to the sensors which is not feasible. So, the solution with the larger range value is considered for such cases. The results are expected to improve significantly if we add one sensor more than the minimum required for a particular dimension (i.e. 4 in 2D and 5 in 3D), which constitutes a future direction of research.

IV. CONCLUSIONS

The study suggests that the algorithm used is able to give a closed form solution for the system of equations. The localization of a ballistic source in real time is possible using data recorded at multiple sources in a 3-dimensional space. The estimated bearing of the source obtained from the algorithm matches very well with the actual bearing which is the critical parameter in locating a gunshot. Error in inputs due to preprocessing needs to be trimmed as the algorithm is very sensitive to slight changes. The estimation of the range is very inaccurate as can be seen from the results. So, more needs to be done to ascertain the algorithm as useful in finding the range of a gunshot. There is a large scope of improving the present results by using other suitable algorithms and hardware. Gunshots produce shock waves in addition to muzzle blasts if the bullet travels supersonically. In such cases the shock wave information can further enhance the solution obtained from the muzzle blast signature.

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