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Comparison of Lighthill's and Lilley's Acoustic Analogies for Jet Noise Modelling

Shabeeb N. P.* and Aniruddha Sinha†
Department of Aerospace Engineering, Indian Institute of Technology Bombay, Powai, Mumbai, 400076, INDIA

This paper studies the jet noise modelling based on two different approaches - Lighthill's and Lilley's. Lighthill's approach is an exact acoustic analogy, while Lilley's approach is an approximate one with an additional assumption of a locally-parallel base flow. Both approaches are based on rearranging the flow equations into a linear propagation operator and a noise source. The noise propagation is determined by calculating the Green's functions of the corresponding linear wave-like propagation operators. In both models, the sources are described as a statistical model of the two-point cross-correlation of the velocity fluctuations, which are characterized by the turbulent length and time scales. To predict the far-field noise, the two models are used for an isothermal and a heated ideally-expanded Mach 1.5 round jets. Lilley's model results demonstrate a good match with the direct predictions from the time-resolved LES data using the Ffowcs Williams-Hawkings (FW-H) method. On the other hand, Lighthill's model results exhibit a good agreement with the FW-H results at higher polar angles of observer locations, but some under-prediction at lower polar locations.

I. Introduction

JET noise is one of the most challenging fluid mechanics problems that researchers have been working on for the last few decades and it is also one of the loudest noises ever produced by mankind. The introduction of the turbofan engine with a progressively higher bypass ratio has reduced jet noise by a significant amount. Further reduction in jet noise while maintaining other performance characteristics is extremely challenging. A considerable amount of research has been conducted towards developing quieter nozzle designs. The development of such designs is challenging because of the cost constraints and the high complexity of the flow field. The understanding of the flow field and sources of jet noise will guide the development of noise prediction tools and that eventually will lead toward quieter designs.

A typical jet noise prediction methodology has the following elements – the formulation of a linear wave-like propagation operator, the designation of the corresponding noise source, calculation/modelling of these noise sources, and solution of the radiated sound. There are numerous possible choices for decomposing the nonlinear governing equations for flows, written compactly here for flow field vector q as $N(q) = 0$, into a noise source $\mathscr{S}$ and a propagation operator L as $Lq = \mathscr{S}(q)$. If the latter is an exact rearrangement of the original $N(q) = 0$ equation then it is called an acoustic analogy; otherwise it is simply a noise model. The first such theoretical formulation for aerodynamic noise prediction was the work of Lighthill [1]. Lighthill formulated his acoustic analogy by reworking the Navier-Stokes equation (NSE) into an inhomogeneous wave equation by combining the time derivative of the continuity equation with the divergence of the momentum equation. He chose the propagator L as the free-space wave operator (i.e., propagation through a quiescent medium), and the corresponding source was found to have a quadrupolar character. The exact solution to the Lighthill's equation can be determined by evaluating the convolution integral of the Green's function of the free-space wave operator with the corresponding source term. Lilley [2] modified Lighthill's equation by considering the propagation of sound through a locally-parallel medium, as is appropriate for many shear flows, and jets in particular. Unlike Lighthill's analogy, the Lilley's propagation operator accounts for the convection and refraction of the sound waves; in Lighthill's approach these effects are subsumed in the source term. Later, Goldstein [3] proposed a generalized acoustic analogy that was an exact consequence of NSE, considering the propagation of sound through an arbitrary medium. These successive developments were geared towards shifting the burden from modelling of the source $\mathscr{S}$ to solving the propagation operator L.

It is computationally expensive and time-consuming to predict noise from a turbulent jet using Direct Numerical Simulation (DNS) or Large Eddy Simulation (LES). Tam and Auriault [4] proposed a semi-empirical theory to predict the far-field noise from fine-scale turbulence that required only the mean flow velocity, density, turbulent kinetic

*Doctoral student
†Associate Professor, AIAA Member.

energy and dissipation in the near-field region. The much more economical steady Reynolds averaged Navier-Stokes (RANS) simulation sufficed for this purpose. The turbulent statistics in the source region were modelled using the turbulent length, time and velocity scales. The authors showed very accurate noise prediction vis-à-vis experimental measurements over a wide range of jet velocities and temperature ratios for single-stream round jets, especially in the sideline and upstream direction where the fine-scale contribution dominates. Refs. [5] and [6] introduced an acoustic analogy based on the linearized Euler equations (LEE) with no assumptions of fine-scale or large-scale noise sources. The former paper also discussed the similarities in the outputs of Lighthill's approach and Tam and Auriault's method, once consistent assumptions are made for the corresponding source terms. The LEE-based approach was also used in Ref. [7] for a Mach 0.9 jet, where comparisons were made with the noise results from the asymptotic solutions given in Refs. [8] and [9]. Miller [10] presented an acoustic analogy that independently predicted the noise from turbulent mixing and shock interactions based on the LEE. Of late, this methodology has been successfully used to predict the noise from chevron jets and axisymmetric dual-stream jets for a wide range of Mach numbers and temperature ratios; it is employed in the present work too. In parallel, Papamoschou [11] has successfully used Lighthill's approach to model the noise from complex multistream jets, and our research is guided by this too.

Samanta et al. [12] have evaluated the robustness of a Lighthill-like analogy, Lilley-like analogy and Goldstein's generalized acoustic analogy for a two-dimensional mixing layer using a DNS solution. The uniform base flow yields the Lighthill-like analogy and a globally parallel base flow yields the Lilley-like analogy. They found that the analogy formulations with the dominant refraction included in the propagation operator are significantly more robust to errors. In this work, Lighthill's and Lilley's acoustic analogy-based noise prediction methods have been studied by modelling their respective source terms as a function of the local turbulent length and time scales. The turbulent length and time scales are modelled from the time-averaged values of turbulent kinetic energy $\overline{K}$ and dissipation $\overline{\epsilon}$ data using simple scaling laws and empirical coefficients. A similar statistical model has been used to describe the turbulence in the flow for both Lighthill's and Lilley's models. A well-validated LES database [13] comprising of two supersonic round jets – one isothermal and the other heated – are used for the analysis. The time-averaged data required for the two noise prediction models are extracted from the LES database. The accuracy of these models is investigated by comparing the predicted spectra with the sound propagated directly from the time-resolved LES data using the Ffowcs Williams - Hawkings (FW-H) approach. Both the models are showing excellent match towards the sideline angles of the jet; however, Lighthill's approach is under-predicting the sound at the louder aft angles compared to Lilley's model.

II. Lighthill's Model

Lighthill's acoustic analogy combines the time derivative of the continuity equation with the divergence of the momentum equation. It can be written in Cartesian tensor notation as

$$\frac{1}{a_\infty^2}\frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x_i \partial x_i} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \tag{1}$$

where a_∞ is the ambient speed of sound and the Lighthill stress tensor T_{ij} is $\rho u_i u_j + (p - a_\infty^2 \rho)\delta_{ij} - \tau_{ij}$. Here p is the pressure, u_i is the velocity vector, ρ is the density, τ_{ij} is the viscous stress tensor, and $(\cdot)'$ denotes fluctuations from the base (ambient) state.

Morris and Farassat [5] presented two noise prediction schemes based on Lighthill's acoustic analogy. The 'Lighthill's Acoustic Analogy: Model 2' described in Ref. [5] has been followed in this paper. The expression for spectral density of far-field pressure (eqn. (61) in Ref. [5]) contains the two-point-two-time cross-correlation of the Lighthill stress tensor, modelled as

$$\left\langle T_{ij}(\boldsymbol{x}_s, t) T_{kl}(\boldsymbol{x}_s + \boldsymbol{\eta}, t + \tau) \right\rangle = A_{ijkl} \exp\left\{ -\frac{|\eta_x|}{\overline{u}_x \tau_s} - \frac{(\eta_x - \overline{u}_x \tau)^2}{\ell_x^2} - \frac{\eta_y^2}{\ell_y^2} - \frac{\eta_z^2}{\ell_z^2} \right\}, \tag{2}$$

where $\boldsymbol{x}_s$ is the source position vector, $\boldsymbol{\eta} := (\eta_x, \eta_y, \eta_z)$ is the spacial separation vector, τ is the time lag, τ_s is the turbulent time scale, $\{\ell_i\}_{i\in\{x,y,z\}}$ are the turbulent length scales, $\overline{(\cdot)}$ denotes the time-averaged quantity and A_{ijkl} is the correlation amplitude. The model posits that the correlation has a Gaussian decay in the spacial direction with different length scales in the axial and cross-stream directions. Since the turbulence is not isotropic because of the convection of the eddies downstream, the axial extent of the correlation must be larger compared to the cross-stream direction. The $(\eta_x - \overline{u}_x \tau)$ term accounts for the eddy convection in a fixed frame of reference, invoking the frozen field hypothesis. Finally, it posits an exponential decay of correlation in time, with time scale τ_s, if one were to move with the mean flow.

Morris and Farassat [5] simplified the spectral density equation for an observer at $\Theta = 90^0$, where Θ is the polar angle of the observer measured w.r.t. the jet downstream axis. That is, the dot product between the spacial lag ($\boldsymbol{\eta}$) and the position vector of the observer ($\boldsymbol{x}$) was neglected (Eqn. (66) in Ref. [5]). Since the current work is not restricted to $\Theta = 90^0$, the spectral density equation can be written as

$$S_p(\boldsymbol{x}, \omega) = \frac{\pi^{-5/2}\omega^4}{32\rho_\infty a_\infty^5 R^2} \int_{\boldsymbol{x}_s} \int_{\boldsymbol{\eta}} \beta_i\beta_j\beta_k\beta_l A_{ijkl} \frac{\ell_x}{\overline{u}_x} \exp\left\{ -\frac{\omega^2 \ell_x^2}{4\overline{u}_x^2} - \frac{i\omega(\boldsymbol{\eta}\cdot\boldsymbol{x})}{R a_\infty} - \frac{|\eta_x|}{\overline{u}_x \tau_s} - \frac{i\omega\eta_x}{\overline{u}_x} - \frac{\eta_y^2}{\ell_y^2} - \frac{\eta_z^2}{\ell_z^2} \right\} d\boldsymbol{\eta} d\boldsymbol{x}_s, \quad (3)$$

where β_i are the direction cosines, ω is the circular spectral frequency of interest, and R is the radial position of the observer in polar coordinates. Performing the integral over $\boldsymbol{\eta}$ analytically, the spectral density equation for an observer located at $\boldsymbol{x} := (R, \Theta, \phi = 0)$ can be written as

$$S_p(\boldsymbol{x}, \omega) = \frac{\pi^{-3/2}\omega^4}{16\rho_\infty a_\infty^5 R^2} \int_{\boldsymbol{x}_s} \beta_i\beta_j\beta_k\beta_l A_{ijkl} \frac{\ell_x \ell_y \ell_z \tau_s}{\omega^2 \tau_s^2 \left(1 + \frac{\overline{u}_x \cos\Theta}{a_\infty}\right)^2 + 1} \exp\left\{ -\frac{\omega^2}{4a_\infty^2} \left(\frac{a_\infty^2 \ell_x^2}{\overline{u}_x^2} + \ell_y^2 \sin^2\Theta \right) \right\} d\boldsymbol{x}_s. \quad (4)$$

The direction cosines for the far-field observer are

$$\beta_1 = \cos\Theta, \quad \beta_2 = \sin\Theta\cos\phi, \quad \beta_3 = \sin\Theta\sin\phi. \quad (5)$$

The amplitude of the two-point-two-time cross-correlation (A_{ijkl}) is modelled in terms of the shear-noise and self-noise [14]. The shear noise is directive because it is produced jointly from turbulence and jet mean flow. Papamoschou [11] has explained each term in shear noise and its modelling. The self-noise is omni-directional because it is assumed to arise from isotropic turbulence [14]. The total contribution from self and shear noise in the direction of the far-field observer is modelled as

$$\beta_i\beta_j\beta_k\beta_l A_{ijkl} = A \left[\underbrace{\overline{\rho}^2 \overline{u}_x^2 \left(\frac{8}{3}\overline{K}\cos^2\Theta + 8g\cos^3\Theta\sin\Theta\cos(\phi_s - \phi) \right)}_{\text{shear noise}} + \underbrace{\overline{\rho}^2 u_s^4}_{\text{self noise}} \right], \quad (6)$$

where A is a constant, $\overline{K}$ is the time-averaged value of turbulent kinetic energy, ϕ_s is the azimuthal location of the source, u_s is the velocity scale and

$$g = \nu_T \left| \frac{\partial \overline{u}_x}{\partial r} \right|,$$

where ν_T is the turbulent viscosity. The second term in the shear noise component in eqn. (6) does not contribute anything to the far-field noise for an axisymmetric jet [14]. The azimuthal integral over this term goes to zero in the spectral density equation.

The turbulent viscosity, turbulent length, time and velocity scales are modelled as [4–7, 10, 11]:

$$\nu_T = c_\mu \frac{(\overline{K})^2}{\overline{\epsilon}}, \quad \ell_x = c_\ell \frac{(\overline{K})^{3/2}}{\overline{\epsilon}}, \quad \tau_s = c_\tau \frac{\overline{K}}{\overline{\epsilon}}, \quad u_s = \sqrt{\frac{2}{3}\overline{K}} \quad (7)$$

where $\overline{\epsilon}$ is the time-averaged value of turbulent dissipation and c_μ, c_ℓ and c_τ are empirical constants. Further, the cross-stream length scales are assumed to be one-third of the streamwise length scales at all locations:

$$\ell_y = \ell_z = \ell_x/3. \quad (8)$$

The turbulent length scale is expected to depend on the spectral frequency being considered. Let us denote the frequency-dependent streamwise length scale as $l_x(\boldsymbol{x}_s, St)$, where $St = \omega D_j/(2\pi U_j)$ is the Strouhal number corresponding to the frequency ω under consideration, D_j is the jet's nozzle-exit diameter, and U_j is its nozzle-exit velocity. Following Ref. [15], all these length scales are modelled as

$$l_i(\boldsymbol{x}_s, St) = \ell_i(\boldsymbol{x}_s) \frac{1 - \mathrm{e}^{-c_f St}}{c_f St}, \quad \forall i \in \{x, y, z\}, \quad (9)$$

where $c_f = 11.25$ was chosen to match the experimental observations. While computing the spectral density, all the length scales are replaced by the frequency-dependent length scales in eqn. (4).

III. Lilley's Model

Our mathematical formulation of Lilley's model is based on the works of Refs. [4–7, 10]. The governing equations are the Euler equations as viscous effects are deemed unimportant for both sound generation and propagation. The equations are

$$\frac{D\pi}{Dt} + \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0, \tag{10a}$$

$$\frac{D\boldsymbol{u}}{Dt} + a^2\boldsymbol{\nabla}\pi = 0. \tag{10b}$$

where $D(\cdot)/Dt$ is the material derivative, a is the local speed of sound and $\pi := (1/\gamma)\ln(p/p_\infty)$ is the logarithmic pressure. The Euler equations are linearized by expanding the flow variables as fluctuations on a time-averaged base state and retaining terms on the left-hand side (LHS) that are linear in the fluctuations while gathering all remaining nonlinear terms on the right-hand side (RHS). The nonlinear terms on the RHS are noise sources. The linearized Euler equations (LEE) are solved by finding its vector Green's function and convolving it with the sources.

Considering a locally parallel mean flow for a round jet, the LEE has been simplified to the Lilley's equation [2]. The Lilley's operator acting on the logarithmic pressure component of vector Green's function of the LEE ($\widehat{\pi}_g^n$) in the Fourier domain can be written as

$$\begin{aligned} L_L\widehat{\pi}_g^n(\boldsymbol{x}|\boldsymbol{x}_s;\omega) &= \overline{D}_\omega^2\left(\delta(\boldsymbol{x}-\boldsymbol{x}_s)\right)\delta_{0n} - \overline{D}_\omega\frac{\partial}{\partial x}\delta(\boldsymbol{x}-\boldsymbol{x}_s)\delta_{xn} - \left[\frac{1}{r}\overline{D}_\omega\frac{\partial}{\partial r}\left(r\delta(\boldsymbol{x}-\boldsymbol{x}_s)\right) - 2\frac{d\overline{u}_x}{dr}\frac{\partial}{\partial x}\delta(\boldsymbol{x}-\boldsymbol{x}_s)\right]\delta_{rn} \\ &\quad - \frac{1}{r}\overline{D}_\omega\frac{\partial}{\partial\phi}\delta(\boldsymbol{x}-\boldsymbol{x}_s)\delta_{\phi n} =: \mathscr{S}^n(\boldsymbol{x}-\boldsymbol{x}_s;\omega), \end{aligned} \tag{11}$$

where

$$L_L = \left(\overline{D}_\omega^3 - \overline{a^2}\,\overline{D}_\omega\nabla^2 - \frac{d\overline{a^2}}{dr}\overline{D}_\omega\frac{\partial}{\partial r} + 2\overline{a^2}\frac{d\overline{u}_x}{dr}\frac{\partial^2}{\partial x\partial r}\right) \tag{12}$$

is Lilley's wave operator and $\overline{D}_\omega := -i\omega + \overline{u}_x\partial/\partial x$. Here, the governing equation is written in cylindrical coordinates because we are considering an axisymmetric jet. The vector Green's functions are indexed by n, which takes values in $\mathscr{N} := \{0, x, r, \phi\}$ corresponding to forcing of the conservation equations for mass and three cylindrical components of momentum. Let the scalar Green's function of the above Lilley's equation be denoted as $\widehat{g}(\boldsymbol{x}|\boldsymbol{x}_s;\omega)$. Then the pressure component of the vector Green's function of LEE can be written as the convolution integral

$$\widehat{\pi}_g^n(\boldsymbol{x}|\boldsymbol{x}_s;\omega) = \iiint \widehat{g}(\boldsymbol{x}|\boldsymbol{x}_t;\omega)\,\mathscr{S}^n(\boldsymbol{x}_t-\boldsymbol{x}_s;\omega)d\boldsymbol{x}_t. \tag{13}$$

The Green's function of the Lilley's equations are solved numerically using the adjoint approach, as explained by Tam and Auriault [16] and Raizada and Morris [7]; subsequently, the pressure component of the vector Green's functions of LEE are determined using eqn. (13). The far-field pressure fluctuations may be obtained by convolving the four Green's functions' pressure components with the corresponding sources of the LEE. We are interested in the spectral density of the pressure and we have followed the work of Miller [10] to determine the spectral density from the governing equations. The two-point–two-time cross-correlation of the nonlinear source terms of the LEE has been modelled using the local turbulent length and time scales, and is of the form

$$\langle f_n(\boldsymbol{x}_s,t)f_{n'}(\boldsymbol{x}_s+\boldsymbol{\eta},t+\tau)\rangle = \delta_{nn'}A_n(\boldsymbol{x}_s)\exp\left\{-\frac{|\tau|}{\tau_s} - \frac{(\eta_x-\overline{u}_x\tau)^2}{\ell_x^2} - \frac{\eta_y^2}{\ell_y^2} - \frac{\eta_z^2}{\ell_z^2}\right\}, \tag{14a}$$

$$A_0 = B_0^2\frac{(u_s/a_\infty)^4}{\tau_s^2}, \qquad A_n = B_{>0}^2\frac{(u_s/a_\infty)^2u_s^4}{\ell_x^2}, \tag{14b}$$

where u_s is the velocity scale and f_n are the nonlinear sources of the LEE; specifically, f_0 is the unsteady dilatation and the other three are the components of unsteady force per unit mass. The model assumes that the four source terms in the LEE are uncorrelated and the magnitudes of the correlation functions are related by dimensional analysis to the local turbulent velocity, time and length scales. The Gaussian terms in this correlation model are the same as that of eqn. (2). Here, the first term in the exponential is slightly different from eqn. (2), where the $\eta_x/\overline{u}_x$ term is replaced by τ.

LES Case	M_{jet}	T_{jet}/T_{∞}	M_a	Re_{jet}
B118	1.5	1.0	1.5	300,000
B122	1.5	1.74	1.98	155,000

Table 1 **Test cases used from LES database of Ref. [13].**

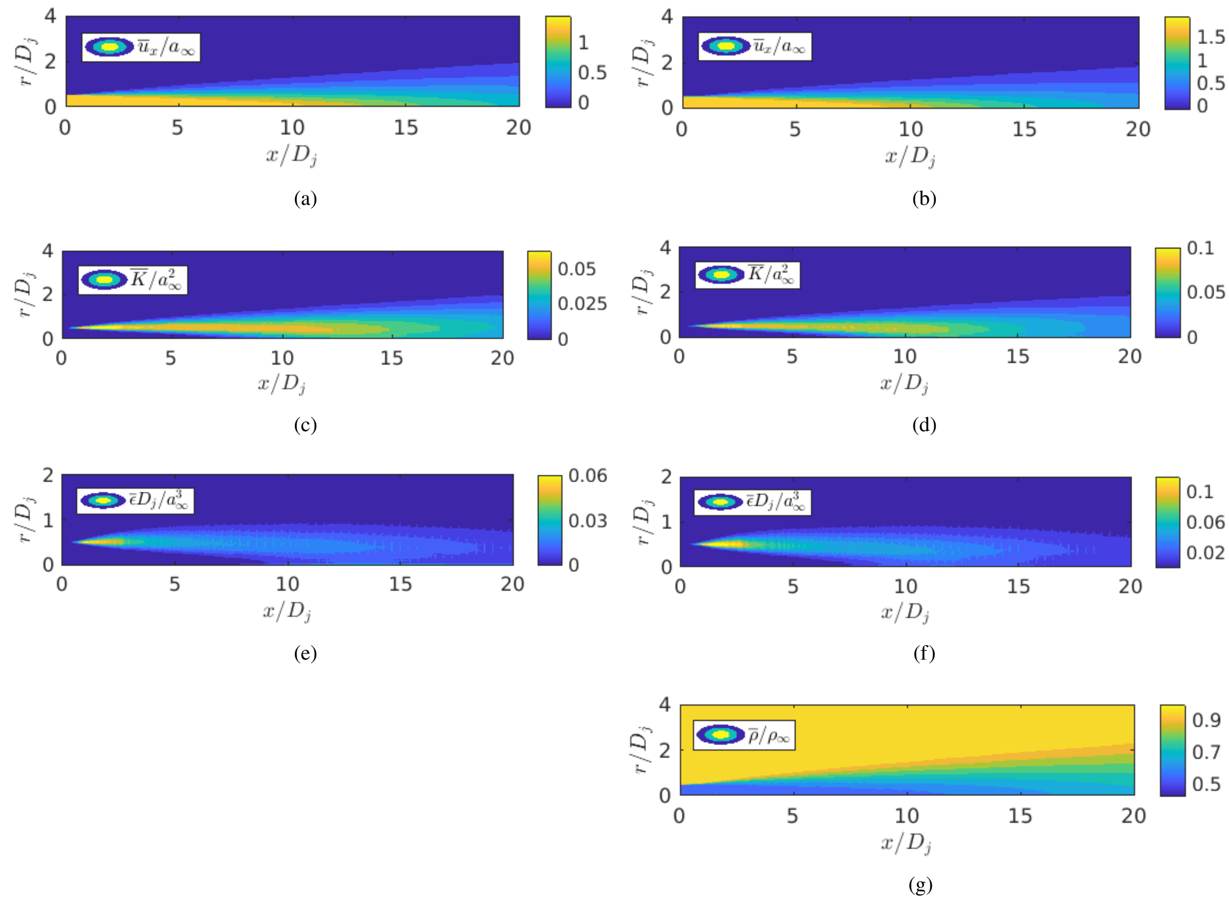

Fig. 1 **Contours of mean values of streamwise velocity of (a) B118 and (b) B122 jets, turbulent kinetic energy of (c) B118 and (d) B122 jets, turbulent dissipation in (e) B118 and (f) B122 jets, and density of (g) B122 jet.**

Assuming the convection of the turbulent eddies at a speed of mean flow velocity, the exponential terms of both the cross-correlation models are the same.

Assuming that the observer is in the far field and the vector Green's function of two closely-placed source points differ by only a phase factor [4], we find the spectral density equation for an observer located at $\boldsymbol{x} := (R, \Theta, \phi = 0)$ as,

$$\frac{S_p(\boldsymbol{x}, \omega)}{(\gamma p_\infty)^2} = 2\pi^{3/2} \sum_{n \in \mathcal{N}} \int \left| \widehat{\pi}_g^n (\boldsymbol{x} | \boldsymbol{x}_s; \omega) \right|^2 \sigma_n(\boldsymbol{x}_s; \omega, \boldsymbol{x}) d\boldsymbol{x}_s,$$
$$\sigma_n := A_n \ell_x \ell_y \ell_z \tau_s \frac{\exp\{-\omega^2(\ell_x^2 \cos^2\Theta + \ell_y^2 \sin^2\Theta)/4a_\infty^2\}}{1 + \omega^2 \tau_s^2 (1 - \overline{u}_x \cos\Theta / a_\infty)^2}. \tag{15}$$

Here also, the turbulent length, time and velocity scales are computed from the time-averaged values of turbulent kinetic energy and dissipation using eqn. (7). Similar to the previous model, consistent assumptions have been used in the cross-stream length scale model (eqn. (8)) and the frequency-dependent length scale model (eqn. (9)).

IV. Results and Discussion

A steady RANS solution is enough to predict the far-field jet noise using the models explained in the previous section. However, if we want to *validate* these models, then we need an independent prediction of the noise, which is impossible with the RANS data. Instead of using a steady RANS solution, the LES results of Brès et al. [13] are used, and the required input parameters are computed from it for the two noise prediction models. The database is summarized in Table 1 and consists of an isothermal ideally-expanded round jet (case B118) and a heated ideally-expanded round jet (case B122). The unstructured LES grid had 42 million control volumes; for the current analysis, this data was interpolated to a cylindrical structured grid having about 1.3 million points, and extending $20D_j$ in the streamwise direction and $3.5D_j$ in the radial direction.

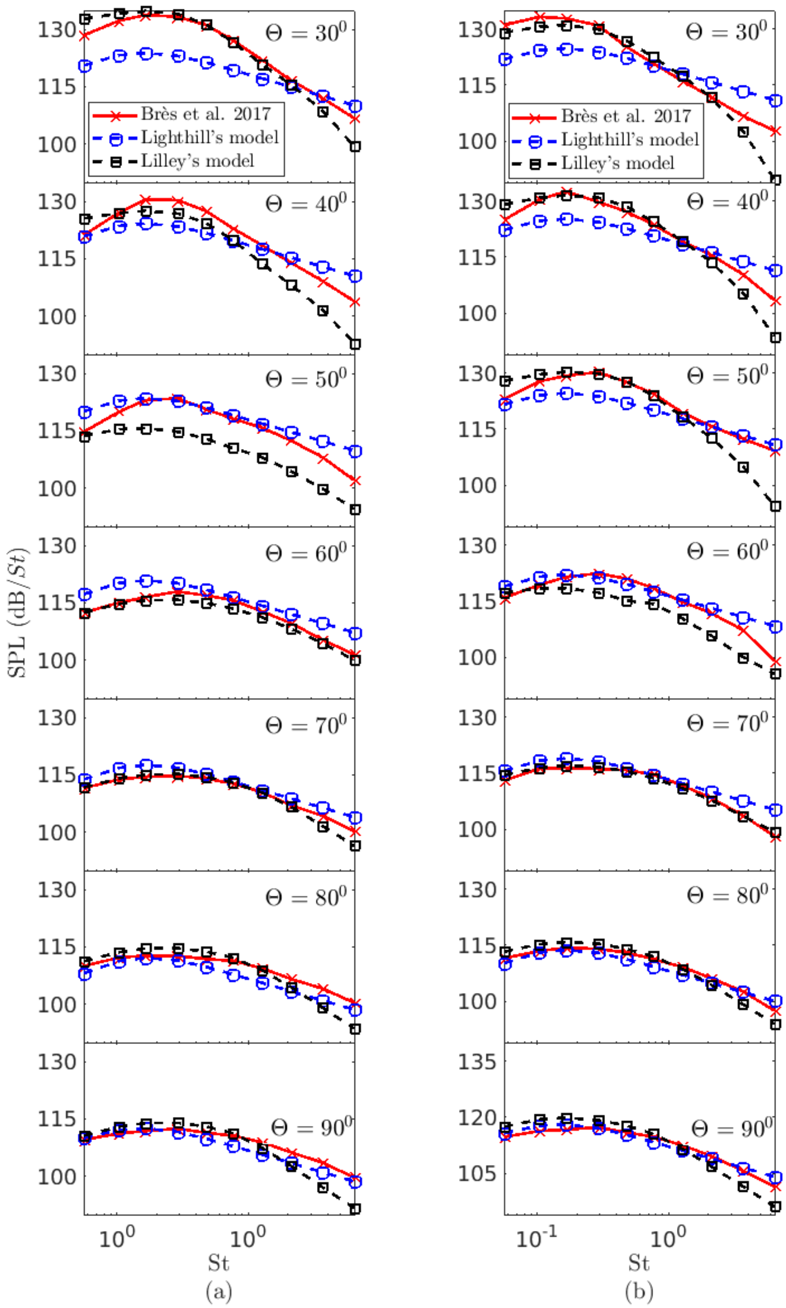

Fig. 2 **Comparison of the far-field noise predicted with the FW-H results for the (a) B118 and (b) B122 jets.**

The input data required for the noise prediction models are the mean flow velocity $\overline{u}_x$, density $\overline{\rho}$, turbulent kinetic energy $\overline{K}$ and dissipation $\overline{\epsilon}$. These quantities are computed from the LES database for both the isothermal and heated jets. The contours of mean flow data extracted from the LES database are shown in Figure 1. The input parameters are nondimensionalized with respect to the nozzle-exit diameter D_j, the ambient density ρ_∞ and the ambient speed of sound a_∞. We can see that the turbulent kinetic energy and dissipation are dominant in the shear layer where the noise sources are active and they are vanishing beyond the inner and outer edges of the shear layer. The noise source models in eqns. (2) and (14) are also showing that the noise sources are active were turbulent kinetic energy and dissipation are active. The density plot of the isothermal B118 jet is not shown, as it is essentially uniform throughout.

The far-field noise is quantified using the sound pressure level (SPL) spectra at various observer locations. The polar angles of these observer locations are ranging from $\Theta = 30^0$ to 90^0 and the corresponding polar radii are chosen to match the location of microphones in the reference experiments of Schlinker et al. [17], wherein a rectilinear array was used. The reference noise spectra presented in fig. 2 for the two jets are calculated directly from the time-resolved flow field fluctuation data available in the LES solutions of Brès et al. [13]. For this, the Ffowcs Williams and Hawkings (FW-H) method [18] is used, as was done in Ref. [13]. The authors reported an excellent match with the reference spectral data from the experiments of Ref. [17], which validated their LES simulations.

The far-field noise predicted using Lighthill's and Lilley's models are plotted along with the reference spectra in fig. 2. It is evident that the Lighthill's model predictions are excellent at higher polar angle observer locations (i.e., towards the sideline angles). But the model is under predicting as the observer location moves towards the lower polar angles (i.e., towards the aft angles). The Lilley's model prediction is showing a good match with the reference spectra except at high frequencies for both jets. Although there are slight differences, both models are agreeing well with the reference spectra at sideline angles. This is because the convection and refraction effects are not having a significant contribution to these observers. The Green's function of the Lighthill's model is the free space Green's function, which accounts for the propagation of the sound. The source terms contain all the convection and refraction effects in the flow field and these are modelled as explained in section II. But in the case of Lilley's model, the Green's function of Lilly's equation consists of both the convection and refraction effects and which is solved numerically. So there is less burden on the modelling of the sources compared to the Lighthill's model.

V. Conclusions

This paper studies the jet noise modelling based on Lighthill's and Lilley's approaches. Lighthill's is an exact acoustic analogy, whereas Lilley's is an approximate one, with the additional assumption of a locally-parallel base flow. Barring this, both are rearrangements of the flow equations into a linear propagation operator and a noise source. The noise propagation is calculated by finding the Green's functions of the corresponding linear wave-like propagation operators. In both models, the sources have been described as a statistical model of the two-point cross-correlation of the velocity fluctuations, characterized by the turbulent length and time scales. The far-field noise has been predicted using the two models for an isothermal and a heated ideally-expanded Mach 1.5 round jets. Lilley's model results show a good match with the direct predictions from the time-resolved LES data using the Ffowcs Williams-Hawkings (FW-H) method. The Lighthill's model results show a good agreement with the FW-H results at higher polar angles of observer locations, but some under-prediction at lower polar locations.

The results above suggest that moving the burden of modelling the sources to solving the wave operator is having a significant effect on the noise prediction especially at lower polar observer locations. In Lighthill's model, the propagation operator is exact and the sources are modelled. Whereas, in Lilley's model, the propagation operator is not exact because of the locally parallel mean flow assumption. Since this assumption is a valid one for a slowly spreading jet, the propagation operator of Lilley's equation is nearly perfect. So we can conclude that the difference in the noise spectra between the two schemes is because of the choice of the model used in the cross-correlation of sources. That means there is a scope for improvement in the modelling of correlation terms in Lighthill's model. The correlation model used in eqn. (2) consists of the amplitude of correlation (A_{ijkl}) and exponential terms that define both the convection and decay of turbulent eddies [19]. Here, the exponential terms do not contain any directivity terms, whereas each of the correlation terms are being multiplied by the corresponding direction cosines. So, there is a scope for extending this work by improving the correlation amplitude of the noise sources used in Lighthill's model for better noise prediction at lower polar locations.

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