Parabolized Stability Analysis of Jets Issuing from Serrated Nozzles

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Abstract Jets issuing from serrated nozzles have a correspondingly serrated time-averaged flow field. We solve the mildly non-parallel linear parabolized stability problem for such high speed turbulent jets to model the coherent wavepackets in the flow. The base flow for the analysis is the mean flow field from a large-eddy simulation database of a cold Mach 0.9 fully turbulent jet issuing from a nozzle with six serrations, a benchmark case in the literature. The fluctuation data is also filtered to extract the most-energetic coherent part using proper orthogonal decomposition. Such filtered data is shown to bear an encouraging resemblance with the predicted wavepackets.

1 Introduction

Large-scale coherent structures have been detected in unforced high-speed turbulent jets issuing from round nozzles several decades ago (Mollo-Christensen 1967). Since then, researchers have proposed models for these structures, mostly based on hydrodynamic stability analysis of the turbulent mean flow field (e.g. Tam and Chen 1994). More recent work (e.g. Gudmundsson and Colonius 2011) have presented detailed validation results for such models by suitable filtering of the wealth of flow data that has become available, both from experiments and from large-eddy simulation (LES). These developments have been reviewed in Ref. (Jordan and Colonius 2013).

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Serrations of the nozzle exit (called chevrons) have been demonstrated to grant appreciable acoustic benefit for high-speed jets (Bridges and Brown 2004). The chevrons are designed to impinge on the jet shear layer, so that the flow field assumes a corresponding serrated nature. The time-averaged flow field of the latter configuration, especially the streamwise vorticity therein, has been studied in depth (e.g. Alkislar et al. 2007). The coherent structures existing in such serrated jets have not been analyzed at length, an exception being the work in Ref. (Gudmundsson and Colonius 2007) that modeled them using linear stability theory (LST).

Parabolized stability equations (PSE) are an improvement over LST whereby mildly non-parallel base flows in convectively unstable flows can be addressed at little additional computational cost (Herbert 1994). Linear PSE has been applied successfully to predict the coherent wavepackets extracted from data of round jets (Cavalieri et al. 2013a; Gudmundsson and Colonius 2011; Sinha et al. 2014). In the present work, we extend linear PSE to model the wavepackets in jets issuing from serrated nozzles.

As in classical stability theory, PSE starts by decomposing the instantaneous flow field vector $\boldsymbol{q}$ into a time-invariant base flow (herein the turbulent mean flow) $\overline{\boldsymbol{q}}$ and the residual fluctuations $\boldsymbol{q}'$ that have the wavepacket-like ansatz

$$\boldsymbol{q}'(x, r, \theta, t) = \breve{\boldsymbol{q}}_\omega (x, r, \theta)\, \mathrm{e}^{-\mathrm{i}\omega t} + \text{c.c.}, \quad \breve{\boldsymbol{q}}_\omega = \sum_{m=-\infty}^{\infty} \tilde{\boldsymbol{q}}_{\omega,m}(x, r)\, \mathrm{e}^{\mathrm{i}m\theta + \mathrm{i}\int_{x_0}^{x} \alpha_\omega(\xi)\mathrm{d}\xi}. \quad (1)$$

Here, $\tilde{\boldsymbol{q}}_{\omega,m}$ is a shape function and α_ω is a complex axial wavenumber, both assumed to have mild axial variation, commensurate with the mildly non-parallel base flow. Substution of (1) in the linearized governing equations, and invocation of the slowly-varing wavepacket assumption yields an approximately parabolic system of linear PDE's (in x and r) that may be marched downstream, as detailed in Ref. (Gudmundsson and Colonius 2011; Sinha et al. 2014).

The serrated nozzles typically have L chevrons distributed *uniformly* around the azimuth. Ref. (Gudmundsson and Colonius 2007) presented a method to exploit the corresponding symmetries of the serrated mean flow field to simplify the LST-based analysis. The same procedure is employed herein, but for the PSE model. The analysis shows that Fourier azimuthal modes $m \in \{0, \pm L, \pm 2L, \dots\}$ form a coupled system, which we call 'azimuthal order' $M = 0$. Analogously, the set of coupled modes $m \in 1 + \{0, \pm L, \pm 2L, \dots\}$ constitutes azimuthal order $M = 1$, and so on.

The PSE predictions are validated against an LES database that simulates a Mach 0.9 cold jet issuing from the benchmark chevron nozzle SMC001 having $L = 6$ chevrons. The original simulation (Xia et al. 2009) had 12.5 million grid cells, and demonstrated favorable agreement of the mean velocity and Reynolds stresses with corresponding NASA experimental data (Bridges and Brown 2004). The grid has since been improved with a doubled azimuthal resolution and 20 million grid cells in total.

2 Results

In this section, we will report the circular frequency ω in terms of the corresponding Strouhal number St. Also, all linear dimensions are implicitly normalized by the nozzle exit diameter. The post-processing of the LES data is similar to that described in Ref. (Sinha et al. 2014) for round jet LES data.

Figure 1 shows the nature of the PSE eigensolution for a representative frequency ($St = 0.3$). Although 6-fold azimuthally-symmetric structures are evident near the nozzle, the behavior is low-order downstream; e.g., $m = 1$ is dominant in the $M = 1$ solution by $x = 2$. In fact, the structure of the wavepackets evident in the solution is quite similar to those observed in round jets (Gudmundsson and Colonius 2011).

PSE models the most amplified average wavepacket that is, by definition, coherent over the flow domain of the jet. Proper orthogonal decomposition (POD) is a filtering tool that may be used to extract the energetic coherent structure in turbulence for comparison with the modeled wavepackets (Cavalieri et al. 2013a; Gudmundsson and Colonius 2011; Sinha et al. 2014). The pressure component of the data is favoured in this validation exercise, since it reflects the wavepacket character most clearly (Jordan and Colonius 2013; Mollo-Christensen 1967). The domain of the present POD is chosen to be $x \in [1, 10]$, $r \in [0, 5]$. The first POD mode of pressure captures 17 to 30 % of the fluctuation energy, across a range of $St - M$ Fourier modes. Given the large domain that greatly exceeds the integral length scales of the flow, this attests to very significant coherence in the turbulence.

We present a visual comparison of the PSE eigensolutions with the first POD modes of the LES data in representative $St - M$ pairs in Fig. 2. Since the PSE solutions have arbitrary overall amplitude (the equations being homogeneous), they are scaled to match the LES data. Reasonable resemblance is seen between the model predictions and the data. This is both in regard of the wavelengths (and hence phase speeds) as well as the overall amplitude envelopes. In general, though, the PSE solution appears to decay somewhat earlier than what the data suggests. The comparison appears better for the $M = 1$ mode rather than the $M = 0$. All these observations have also been made regarding PSE solutions for round jets (Cavalieri et al. 2013a; Gudmundsson and Colonius 2011; Sinha et al. 2014).

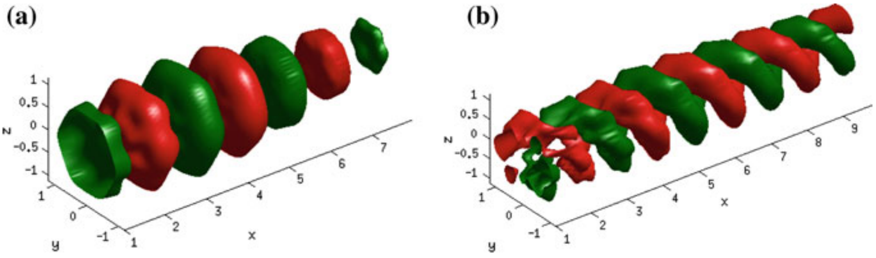

Fig. 1 Representative positive and negative isosurfaces of the real part of pressure in the PSE solution for $St = 0.3$ corresponding to azimuthal orders (**a**) $M = 0$, and (**b**) $M = 1$.

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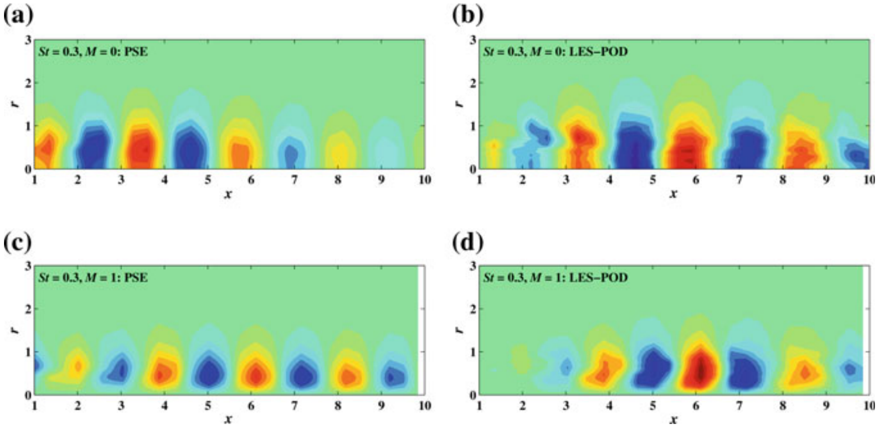

Fig. 2 Least-order azimuthal modes of pressure in PSE solution (*left panels*) compared with corresponding components of first POD mode from LES database (*right panels*) for $St = 0.3$ and azimuthal orders $M = 0$ & 1. Contour levels are identical between each pair of *left* and *right panels*, but not across rows.

A more quantitative comparison is performed using the alignment metric (Sinha et al. 2014)

$$\mathcal{A}_{\omega}^{M} := \left|\left\langle \phi_{\omega}^{M,(1)}, \breve{p}_{\omega}^{M} \right\rangle\right| \left\|\breve{p}_{\omega}^{M}\right\|^{-1} \left\|\phi_{\omega}^{M,(1)}\right\|^{-1}. \tag{2}$$

Here, $\breve{p}_{\omega}^{M}$ represents the pressure component of the PSE solution, and $\phi_{\omega}^{M,(1)}$ is the first POD mode of pressure. The inner product (and induced norm) represent integration over the spatial domain of interest. The above definition implies that $0 \leq \mathcal{A} \leq 1$. A value close to unity indicates that the PSE solution is structurally equivalent (aligned) to the most coherent wavepacket found in the flow.

The alignment metric is reported for various Fourier mode pairs in Table 1. While the qualitative observations made regarding Fig. 2 are borne out, the degree of similarity between the predicted wavepackets and those found the data is re-emphasized here.

Table 1 Alignment metric for PSE solutions vis-à-vis first POD modes of pressure in LES database.

	$St = 0.2$	$St = 0.3$	$St = 0.4$
$M = 0$	0.78	0.78	0.71
$M = 1$	0.91	0.84	0.86

3 Conclusion

We propose a stability theoretic model of the coherent wavepackets present in the turbulent shear layer of jets issuing from serrated nozzles. We apply the PSE model while making explicit use of the statistical symmetries resulting from the symmetries in the nozzle geometry. The base flow field data for the stability analysis is the time-averaged mean flow taken from a validated LES database that simulated a Mach 0.9 cold jet issuing from the benchmark SMC001 nozzle. The PSE results are validated against the fluctuation data, filtered with POD. Encouraging agreement is observed in qualitative and quantitative terms. We conclude that the PSE-predicted instability waves are indeed detectable in turbulent high speed jets issuing from serrated nozzles.

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References

Alkislar MB, Krothapalli A, Butler GW (2007) The effect of streamwise vortices on the aeroacoustics of a Mach 0.9 jet. J Fluid Mech 578:139–169

Bridges JE, Brown CA (2004) Parametric testing of chevrons on single flow hot jets. In: 10th AIAA/CEAS aeroacoustics conference, AIAA Paper 2824

Cavalieri AVG, Rodríguez D, Jordan P, Colonius T, Gervais Y (2013a) Wavepackets in the velocity field of turbulent jets. J Fluid Mech 730:559–592

Gudmundsson K, Colonius T (2007) Spatial stability analysis of chevron jet profiles. In: 13th AIAA/CEAS aeroacoustics conference, AIAA Paper 3599

Gudmundsson K, Colonius T (2011) Instability wave models for the near-field fluctuations of turbulent jets. J Fluid Mech 689:97–128

Herbert T (1994) Parabolized stability equations. In: Special course on progress in transition modelling, AGARD Report No. 793

Jordan P, Colonius T (2013) Wave packets and turbulent jet noise. Annu Rev Fluid Mech 45:173–195

Mollo-Christensen E (1967) Jet noise and shear flow instability seen from an experimenter's point of view. J Appl Mech 34(1):1–7

Sinha A, Rodríguez D, Brès G, Colonius T (2014) Wavepacket models for supersonic jet noise. J Fluid Mech 742:71–95

Tam CKW, Chen P (1994) Turbulent mixing noise from supersonic jets. AIAA J 32(9):1774–1780

Xia H, Tucker PG, Eastwood SJ (2009) Large-eddy simulations of chevron jet flows with noise predictions. Int J Heat Fluid Flow 30:1067–1079

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