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Comparative Study of Azimuthally Dominant Unstable Modes for an Offset Jet at Different Strouhal Number

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Abstract
Coaxial jets are quieter compared to single stream jet. Keeping in mind the importance of the downward sector of the jet, offsetting the secondary jet (with axis parallel to primary jet) can add more thickness to the jet in the required downward sector. Experiments have also shown the noise reduction in the critical downward sector. Recently we have proposed a fitting function suitable for modelling the mean axial velocity field of such offset multi-stream jets. This fitted mean flow field is used in conjunction with a quasi-parallel linear bi-global stability analysis to model the wavepacket noise sources in the jet. We hypothesize azimuthal dominance based on the character of the eigensolutions. In this paper, we report on partial consistency in the directional (azimuthal) dominance of the solution for the same model with partially offset jet when excited with 0.3 and 0.4 Strouhal number (St) compared to 0.1 St.

Keywords : *Linear bi-global stability theory; Turbulent mean flow; Offset jet ; Unstable modes*

I. INTRODUCTION

Coaxial jets are of technological importance in high bypass ratio jet engines. It not only improves cycle efficiency but also leads to reduced jet mixing noise as the bypass stream lowers the jet velocity . Because the aerodynamic noise scales with the jet velocity, this is a very efficient way of decreasing the radiated noise [1]. Increasing bypass ratio will require higher jet diameter which is not feasible to generate required thrust. This motivates to search for other approaches for noise reduction that are feasible under aviation regulations. Another approach is to offset the two jets by keeping the axes parallel [2]. This modifies the azimuthal directivity of the bypass stream so as to thicken the bottom sector of shear layer.

For modeling the noise source, we are interested in computationally efficient approaches, the one pursued here being wave packet theory. Such spatially growing, saturating and decaying structure are prominently found in hydrodynamic near pressure field which exhibits coherence over a significantly larger region compared to the characteristic length of flow. This large scale coherent component of the turbulent jet is mostly responsible for major (loud) part of jet noise radiated at the aft angle [3].

Wave packets in the turbulent jet were initially observed experimentally by Mollo-Christensen [4]. Subsequently, they were modeled as linear Kelvin-Helmholtz (**K-H**) instability wave for time averaged turbulent jet [5]. In the recent years, Linear Stability theory (LST, parallel flow assumption) and Parabolised Stability Equation (PSE, slowly varying flow assumption) have been successfully used to model the wave packets with extensive validation with experiments and large-eddy simulations for high Reynolds number single stream jet [6,7].

In coaxial jets, using spatial LST, two independent modes for a given azimuthal wave number and angular frequency pair have been observed which corresponds to the inner and outer shear layers[8]. They are called as 'inner' and 'outer' mode respectively. Of course these modes can be separately identified before the end of potential core only.

We have already hypothesized that acoustic benefit in offset jet is caused due to favorable azimuthal asymmetry and possibly reduced growth rate of **K-H** modes supported by asymmetric mean flow field. For Strouhal number St = 0.1 (based on primary exit jet diameter and velocity), the inner eigenmodes (that remain unstable over a significant length of the jet) are preferentially dominant in the top sector of the jet. This is hypothesized to be linked to the dominant acoustic radiation of the offset jet to the top sector [12]. In this paper we are trying to assess whether the behavior is replicated at the acoustically-relevant Strouhal range of St = 0.3 − 0.4.

II. MEAN FLOW DESCRIPTION

Our mean axial velocity fields are derived by fitting the data for the coaxial C17M90 and eccentric E17M90 jets studied by Murakami and Papamoschou [2]. The ratio, secondary jet diameter to primary jet diameter (D_s/D_P) is 1.7 with primary Mach number as 1.5 and secondary

Mach number as 0.9. In the eccentric case, the secondary jet was completely offset downwards. All the lengths are normalized by D_p and velocity by primary nozzle exit jet velocity U_p. For all x axial stations, we fitted the velocity profiles with offset double truncated Gaussian by introducing a new parameter 'C' as eccentricity. With this, the mean flow profiles for a concentric jet could be retrieved just by setting 'C'=0; alternatively by setting it to 0.35 we can retrieve fully eccentric jet flow field of E17M90 case[12]. Positive values of 'C' represents downward shift of the secondary jet. With the same functional form, velocity profiles with intermediate eccentricity (between $C = 0$ and 0.35, see fig. 1) can also be obtained.

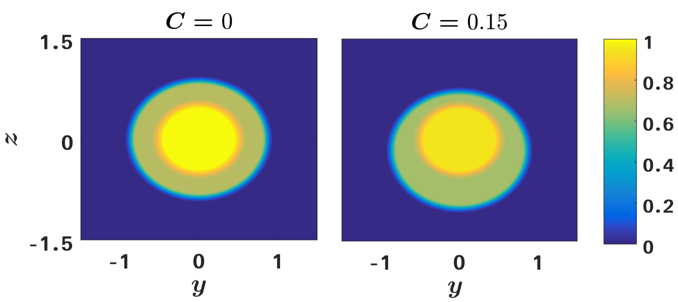


Figure 1: **Mean axial velocity u/U_p contours at x = 1 station**

In the round coaxial jets, the Fourier azimuthal modes are decoupled. The offset jet's mean flow field is inhomogeneous in the azimuth, which couples all azimuthal Fourier modes of the fluctuations. For comparison between coaxial and offset jet results, the LST will be formulated in polar coordinates. Thus, expanding the mean flow axial velocity in azimuthal Fourier modes

$$\bar{u}(x, r, \theta) = \sum_{m=-\infty}^{+\infty} \hat{\bar{u}}_m(x, r) e^{im\theta} \tag{1}$$

where $\hat{\bar{u}}_m$ is the mth azimuthal Fourier mode of $\bar{u}$. Exploiting the mirror symmetry of the offset jet (θ defined with reference to z axis), all positive azimuthal modes are identical to negative azimuthal modes. Let M be the maximum azimuthal mode needed to describe the mean flow by neglecting the contribution of higher order modes.

In the following problem, we have used parallel flow assumption such that the mean radial and azimuthal velocities are negligible. We assumed jet to be isothermal as no information regarding mean temperature (and density) was reported in Ref.2.

III. SPATIAL LINEAR STABILITY THEORY

Neglecting the axial variation of the mean flow and considering it as time invariant, the remaining problem is stated as bi-global stability problem owing to inhomogeneity in r and θ (cylindrical coordinates). Let $\mathbf{q} = \{u, v, w, p, \zeta\}^T$ be the mean flow variable, which denotes axial, radial, azimuthal velocity, pressure and specific volume respectively. In LST, we decompose flow variable into its mean component ($\bar{\mathbf{q}}$) and its fluctuating component ($\mathbf{q}\prime$). The solution ($\mathbf{q}\prime$) is separable in frequency and axial wave number components. Since we do spatial LST, we keep the axial wave number to be complex. Thus, the ansatz will be

$$\mathbf{q}\prime(x, r, \theta) = \hat{\mathbf{q}}_\omega(r, \theta) e^{i(\alpha x - \omega t)} + c.c. \tag{2}$$

$$\hat{\mathbf{q}}_\omega(r, \theta) = \sum_{m=-\infty}^{+\infty} \tilde{\mathbf{q}}_{\omega,m}(r) e^{im\theta} \tag{3}$$

Here, m is the azimuthal Fourier mode, ω is the real frequency described in terms of Strouhal number $St = \omega D_p / 2\pi U_p$. The complex wave number α has real part (α_r) and imaginary part (α_i) which defines as wave number and decay rate (negative of growth rates) respectively.

Substituting the ansatz into the linearized governing equations with mean flow decomposition as described in eq. 1 and taking the azimuthal Fourier transform of the resulting equation for an arbitrary azimuthal Fourier mode n, one arrives at a generalized eigen value problem. The nth azimuthal mode of the solution $\tilde{\mathbf{q}}$ is coupled with other azimuthal modes in the set $[n - M, n + M]$ [12]. Usually, the solution converges with a finite number of azimuthal modes contributing to the solution, say $\pm N$. In coaxial jets, since the azimuthal Fourier modes are independent, the resulting solution can also be characterized with m = 0, 1, 2 and so on. But in case of offset jets, this is not an option due to inhomogeneity in the azimuthal domain. Thus we introduced a new nomenclature μ. When we say μ = 0, it means that eigen solution is dominated by m = 0, for μ = 1, solution is dominated by m =1 and so on.

The grid is clustered in the shear layer where the mean velocity gradient is maximum. The fourth order central difference is use to discretize radial domain. The resulting eigen value problem is sparse, and it is solved using the parallel implementation of the ARPACK library (11) with usual boundary conditions applied to the centerline [9] and the far field [10].

IV. RESULTS

Recently, the effect of offset on the growth rates and eigen function dominance for St = 0.1 has been studied

[12]. In this paper, we have compared the results of St = 0.1 with St = 0.3 and St = 0.4 results at the first axial station where we have mean flow data, viz. x = 1. Here we have presented pressure eigen functions (in r, θ domain) since they are most relevant to acoustics. For the convergence of LST calculations, maximum number of azimuthal mode required to describe the mean flow (azimuthal complexity, M) and for the solution (N) are 25 and 30 respectively. The radial domain (limited to r= 8.5) is discretized with 800 points.

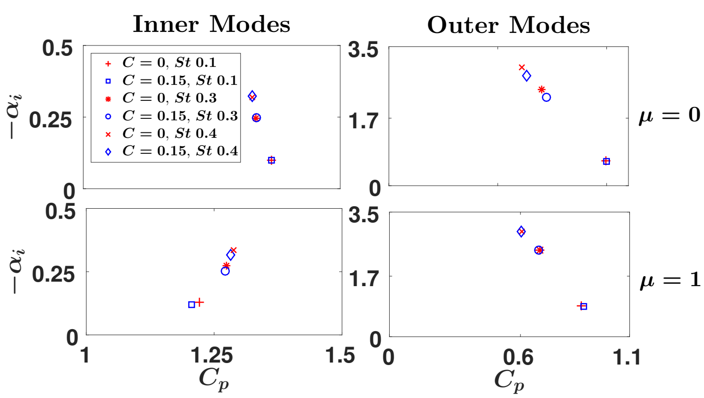


Figure 2: Eigen spectrum for $C = 0$ and $C = 0.15$ cases plotted for three different St (0.1, 0.3 and 0.4). Left column represents inner modes and right column represents outer modes. Only first two dominant azimuthal mode ($\mu = 0$ and 1) traverse is shown for variation in St and C

Fig. 2 shows the eigen spectra for inner and outer modes representing St of 0.1, 0.3 and 0.4, for first two azimuthally dominant modes (μ = 0 and 1). Focusing on inner modes, we observe that for μ = 0, there is no significant change in the growth rates and phase speed between $C = 0$ (coaxial) and $C = 0.15$ (eccentric) cases, for respective St's. However for $\mu = 1$ case, by adding the eccentricity, slight decrease in growth rates can be seen. The offset is seen to slightly stabilize the $\mu = 1$ modes. Further, looking at eigen function plot (see fig. 4(a)), for $St = 0.1$ and $\mu = 0$, eigen functions are dominant at the bottom sector of the jet. This effect is consistent for St = 0.3 and St = 0.4 cases. On the other hand, for μ = 1, even though St 0.1 case is dominant on the upper part of jet, eigen functions for St 0.3 and St 0.4 have distributed strengths on both, upper and bottom sector of jet, with broader spread of eigen function on the upper sector.

Looking at the outer modes (see fig. 2), minor decrease in the growth rates can be observed for μ = 0 compared to μ = 1 by adding the offset for respective St cases. The major differences in growth rates are within different St cases for both azimuthal modes (μ = 0 and 1). Compared to the inner modes, these modes seems to posses higher growth rates as the secondary Mach number is very high (around 0.9) which is not usually the case. So even though, in this study, outer modes seems to be more important, but in actual case, it is the inner modes which are of the prime concern. Contrary to inner modes, we observe that, for St = 0.1, eigen function for $\mu = 0$ dominates on the upper part of the jet (see fig. 4(b)).

For offset jet, the characterization of outer modes with nomenclature as μ for St = 0.3 and St = 0.4 is not straight forward, as we can see that we cannot clearly distinguish outer mode as axisymmetrical or helically dominated by merely comparing with concentric jet cases. For identifying these modes as described above, analysis for intermediate eccentricity (C = 0.05) was conducted (see fig. 3) at St = 0.3 and St = 0.4 (not shown in figure) for tracking the behavior of respective azimuthal mode in respect of azimuthal directivity as well as growth rates.

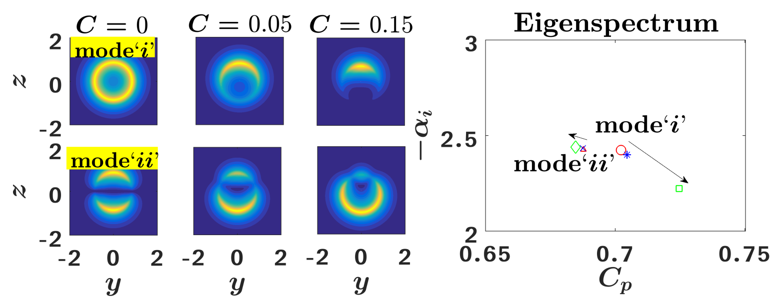
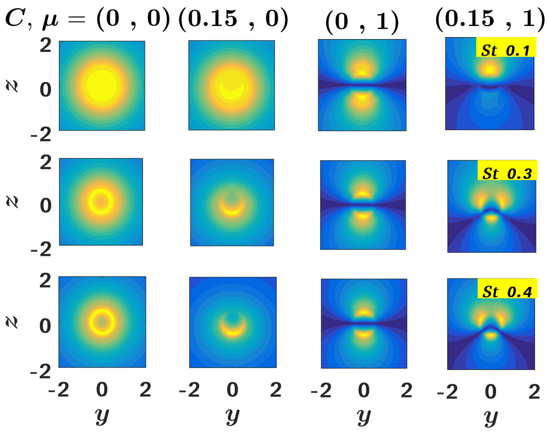


Figure 3: Outer modes pressure eigen function plots for concentric (C = 0), intermediate (C = 0.05) and partially eccentric (C= 0.15) jet at St = 0.3. First row represents mode 'i' and second row represents mode 'ii' showing μ = 0 and μ = 1 mode characteristics respectively. Eigen spectra shows their respective traverse with increasing C

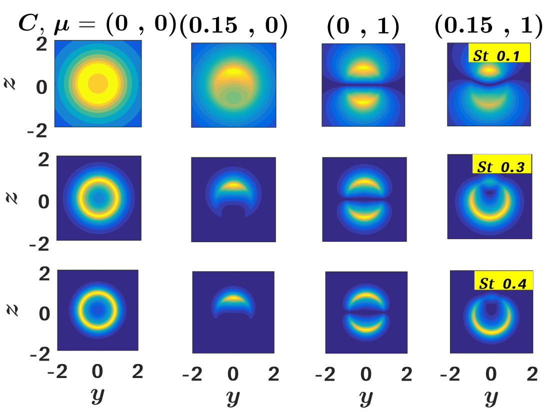
For concentric jet (C = 0), say axisymmetric mode (m = 0) be the mode 'i' and the first helical mode (m = 1) be the mode 'ii'. By offsetting the jet with C = 0.05, we see that the mode 'i' and the mode 'ii' carries the characteristics of axisymmetric mode and first helical mode respectively, with the former being dominant in the upper sector and the later being dominant at the bottom sector. By further offsetting the jet up to C = 0.15, the respective azimuthal dominance is retained by mode 'i' and mode 'ii'. This justifies the use of nomenclature for mode 'i' as μ = 0 and mode 'ii' as μ = 1 at C = 0.15.

Comparing St = 0.3 and St = 0.4 cases with St =

0.1, $\mu = 0$ mode is consistently dominated at downward sector, on the other hand, $\mu = 1$ mode for St = 0.3 and St = 0.4 shows dominance in the downward sector, which is contrary to St = 0.1 case.



(a)



(b)

Figure 4: (a) Inner mode pressure eigen function, (b) Outer mode pressure eigen function plots for concentric (C = 0) and partially eccentric (C= 0.15) jet at St of 0.1, 0.3 and 0.4 (arranged row wise). First two columns represents μ = 0 mode and last two columns represents $\mu = 1$ mode.

V. CONCLUSIONS

Recently we have modeled the noise source for the offset jet at 0.1 Strouhal number (St) [12]. In this paper, we have compared the eigen solution of above stated model with the solution obtained at St 0.3 and St 0.4 with partial eccentricity (C = 0.15) added in the secondary jet (downward jet thickened). Here, we have reported comparison for the modes dominated by axisymmetric (μ = 0) and first helical mode (μ = 1), as the lower order modes are acoustically efficient due to their stronger coherence.

LST results shows that, for all St cases (0.1, 0.3 and 0.4), both inner and outer azimuthal modes dominated by axisymmetric modes shows consistent azimuthal directivity with former showing preference at upward sector and later strengthening at downward sector of the jet. However, at St = 0.3 and 0.4, inner modes dominated by first helical modes partially retains the azimuthal character of eigen solutions compared to St = 0.1 case by showing preference at the upward sector of the jet. Further, at St = 0.1, the outer modes dominated by first helical mode have strong azimuthal preference in the upward sector whereas at St = 0.3 and St = 0.4, these modes have major dominance in the downward sector of the jet.

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