Dynamics & Control in State Space

Introduction

State space methods are concerned with the behaviour of dynamical systems using time domain representation and have evolved from the mathematical study of optimal control. Classical methods, provide limited success when there are multiple inputs, multiple outputs and multiple performance specifications required to be met by the system simultaneously. This is because the classical design methodology considers only single-input single-output forms and also aims to achieve only dominant system response. State space aims to overcome this limitation.

State Space Technique

Basic philosophy of state-space based technique is to arrive at an explicit time domain solution of applicable equations for both open and closed loop systems. The analysis method is based on the premise that an n^{th} order system can be represented as a set of “n” 1^{st} order systems, as shown below.

$$\dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = f(t);\ \text{Let } y = x_1;\ \dot{y} = x_2,\ \ddot{y} = x_3$$

$$\dot{x}_1 = x_2;\ \dot{x}_2 = x_3;\ \dot{x}_3 = -a_3x_1 - a_2x_2 - a_1x_3 + f(t)$$

It is seen that three 1^{st} order equations completely describe the features of the original 3^{rd} order equation.

Linear Systems Based Approach

In general, differential equations describing the behaviour of practical dynamic systems are non-linear, time varying & uncertain and, thus, have no known general close form solutions. However, linear time invariant (LTI) representation is considered an acceptable simplification of the general governing equations, which also offers great advantages in terms of solution simplicity and acceptable accuracy. State-space based treatment of LTI systems also provides an advantage over Laplace domain methods in terms of more accurate treatment of real system effects.

State Space Methodology

As the set of 1^{st} order equations can be manipulated similar to the algebraic equations, solution of these equations can also be manipulated algebraically to provide the solution for the original n^{th} order system. As linear algebraic systems are solved using concepts of vector space and matrix methods, it seems possible that even linear differential equations can be solved using the same, or similar, strategy. Present course aims to apply linear algebraic methods to define a (1) suitable vector space, (2) solution strategy for LTI systems & (3) control options.

Course Objectives

To review linear algebra and matrix methods relevant for state – space based LTI analyses.
To establish applicability of linear algebraic methods to the solution of LTI dynamical systems.
To bring out analogies that exist between the classical and state-space based techniques.
To explore various design methodologies and strategies for closed loop control in state – space.
To demonstrate the applicability of these tools to non-LTI systems, wherever possible.

Course Contents

Time domain form of dynamical systems, time response of higher order linear systems, algebraic perspective for dynamical systems. Review of vector spaces, linear independence, basis vectors, dimension & transformations. Solution of linear algebraic systems, concept of kernel and image spaces, eigenvalues, and eigenvectors. Diagonal and Jordan forms, characteristic equation, operator form, and Cayley-Hamilton theorem. System response in vector space, representation in the state-space, canonical forms. Fundamental matrix and state transition matrix, solution of homogeneous and non-homogeneous systems, matrix exponential. Energy based stability hypothesis, Lyapunov’s theorem of stability, phase plane and state-trajectory based stability analyses. Controllability of dynamical systems. Regulator problem and full state feedback control structure. Pole placement design technique, tracking control structures, optimal control system using Linear Quadratic Regulator (LQR). Output feedback control concept. Observability and its role in control, full and reduced order observers, observer controllers. Kalman filter concept and mechanization, Linear Quadratic Gaussian (LQG). Optimal tracking control. Eigen structure control, H_∞ norm based robust controllers. Stability of nonlinear systems, feedback linearization and nonlinear dynamic inversion (NDI) techniques.

Pre-requisites

While, the course has no specific pre-requisites, participants should be comfortable with linear algebraic and matrix-based methods. Though, the course will touch upon many of these aspects, only those parts that are relevant for understanding state – space, will be dealt with in detail. Further, the course delivery will depend heavily on the MATLAB/SIMULINK as the tool for problem solving and it is expected that participants are fairly comfortable with its usage and syntax.

Text / References

1. Ogata, 'State Space Analysis of Control Systems', Prentice Hall, USA, 1967.
2. Kwakernaak & Sivan, 'Linear Optimal Control Systems', Wiley-Interscience, 1972
3. Kailath, 'Linear Systems', Englewood Cliff: Prentice Hall, 1980.
4. Friedland, 'Control System Design: An Introduction to State-space Methods', McGraw-Hill, 1986.
5. D'azzo & Houpis, 'Linear Control System Analysis & Design…...', McGraw-Hill, 1995.
6. Ogata, 'Modern Control Engineering', 5th Ed., Prentice Hall India, 2010.