Dynamics and Control in State Space

Introduction

State space methods are concerned with the prediction of the behaviour of dynamical systems using time domain representation and have evolved from the mathematical study of optimal control.

Motivation for the Course

In this context, it is worth noting that classical control methods provide limited success when there are multiple inputs and outputs as well as multiple performance specifications, which are required to be met by the system simultaneously. This is because the classical design methodology considers only single-input single-output forms and also aims to achieve only dominant system response. State space based dynamic analyses and control techniques aim to overcome this limitation. Further, it is found that state-space based treatment of systems also provides an advantage over classical methods in terms of more accurate treatment of real system effects. In view of the above, the course presents an overall view of the state-space based techniques for solving dynamical systems and applicable control related tools and procedures, which help those desirous of learning ways of dealing with systems in the physical domain.

Description of State-space Based Methodology

Basic philosophy of state-space based technique is to formulate and solve, the applicable equations for both open and closed loop systems. The analysis method is based on the premise that an n^{th} order system can be represented as a set of "n" 1^{st} order systems. In addition, these set of 1^{st} order equations can be manipulated in a manner similar to the algebraic equations so as to arrive at the solution for the original n^{th} order system. The methodology is particularly suited to the systems that are LTI in nature, because the solutions can be arrived at using the concepts of linear vector space and matrix methods.

Course Objectives

To review linear algebra and matrix methods relevant for LTI system analyses.
To establish applicability of linear algebraic methods to the LTI dynamical systems.
To bring out analogies that exist between the classical and state-space based techniques.
To explore various design methodologies for closed loop control in state – space.
To demonstrate the applicability of these tools to non-LTI systems, wherever possible.

Course Contents

Module-1: Time Domain Modelling - Time domain form of dynamical systems, time response of higher order systems, algebraic perspective for dynamical systems. (**4 Hours**)

Module-2: Review of Vector Spaces - Linear independence, basis vectors, dimension & transformations. Solution of linear algebraic systems, concept of kernel, image and eigen spaces. Diagonal and Jordan forms, characteristic equation, operator form of LTI systems and Cayley-Hamilton theorem. (**8 hours**).

Module-3: System Response in Vector and State-space - Canonical forms of state-space representation. Fundamental matrix and state transition matrix (or matrix exponential), solution of homogeneous and non-homogeneous systems. (**6 Hours**).

Module-4: Stability Analyses - Energy based stability hypothesis, Lyapunov's theorem of stability, phase plane and state-trajectory based stability analyses, Application of Lyapunov's Stability method to nonlinear systems. (**6 Hours**).

Module-5: State Feedback Based Control Structures - Controllability of dynamical systems. Regulator problem and full state feedback control structure. Pole placement design technique, tracking control structures. (**6 Hours**).

Module-6: Output Feedback Based Control Techniques - Output feedback control concept. observability and its role in control, full and reduced order observers, observer controllers. (**4 Hours**).

Module-7: Advanced Control Concepts – Basic optimal control with Linear Quadratic Regulator (LQR) and optimal tracking control. Kalman filter concept and its mechanization. Concept of Linear Quadratic Gaussian (LQG). Eigen structure control and H_∞ norm based robust controllers. (**8 Hours**).

Pre-requisites

While, the course has no specific pre-requisites, participants should be familiar with linear algebraic and matrix-based methods. Though, the course will touch upon many of these aspects, only those parts that are relevant for understanding state – space will be dealt with in detail. Further, it would be useful to have familiarity with the solution of ordinary differential equations, numerical techniques and software systems that help in solving the dynamical equations.

Text / References

Ogata, 'State Space Analysis of Control Systems', Prentice Hall, USA, 1967.
Kwakernaak & Sivan, 'Linear Optimal Control Systems', Wiley-Interscience, 1972
Kailath, 'Linear Systems', Englewood Cliff: Prentice Hall, 1980.
Friedland, 'Control System Design: An Introduction to …..', McGraw-Hill, 1986.
D'azzo & Houpis, 'Linear Control System Analysis & Design', McGraw-Hill, 1995.
Ogata, 'Modern Control Engineering', 5th Ed., Prentice Hall India, 2010.

Mode of Conduct and Delivery

The course is planned to be conducted fully in the on-line format, with two hours per week engagement. Further, it will be run in a module-wise sequence.