System Modeling, Dynamics and Control

Tutorial 1, Autumn Semester, 2006

First-Order Systems

1. The change in tension per unit length of a rope wound around a rough cylinder is proportional to the tension. The constant of proportionality is given by μ/R, where μ is the coefficient of friction and R is the radius of the cylinder. If $\mu = 0.35$, then through what angle must a rope be wound around a cylinder of diameter 30cm so that a person holding one end of the rope can resist a force that is 200 times greater than the force the person exerts?

2. The time constant of a series RC circuit is the time that the capacitor takes to discharge to e^{-1} of its initial charge when the circuit is closed. Find an expression for the time constant in terms of the resistance R and capacitance C. Show that the time constant is also the time that the capacitor would take to disharge completely were it to discharge at a constant rate equal to the initial rate of discharge. What is the time taken by the capacitor to discharge to 50% of its initial value?

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Tutorial 2, Autumn Semester, 2006

Second-Order Systems

1. A thin board of weight W is supported on two identical cylindrical rollers rotating in opposite directions at the same angular speed. If the coefficient of friction between the board and the rollers is μ, describe the motion of the board after its center of gravity is initially displaced by a small amount. Assume that the rollers are of radius r, the distance between their centers is $2c > 2r$ and their angular speed is ω. Does your answer depend on the fact that the initial displacement is small?

2. A locomotive of mass 2000 kg travelling at a speed of 10 m/s is to be stopped at the end of the tracks by a spring-damper system. The spring has a stiffness 40 N/m and the dashpot has a damping coefficient of 20 Ns/m. What is the minimum stroke (range of motion) that the dashpot must have?

3. The barrel of a cannon translates against a recoil mechanism consisting of a spring-damper system. The recoil mechanism is critically damped so that the gun barrel returns to its position without oscillating. The mass of the barrel and the recoil mechanism is 500 kg and the recoil spring has stiffness 10,000 N/m. If the barrel is found to recoil 0.4 m upon firing, find the damping coefficient of the recoil mechanism and the recoil velocity of the gun.

4. The ratio of two successive amplitudes of a viscously damped single degree of freedom system is found to be 18 : 1. Determine the ratio of successive amplitudes if the damping ratio ζ is halved.

5. The indicator mechanism of a DC voltmeter is to be designed such that the pointer has an underdamped response with a peak overshoot of 15% and a rise time of 0.15 s. Moreover, the pointer should settle to within 2% of its final value in less than 0.35 s. Calculate the damping ratio and the natural frequency of the indicator mechanism. You may use $4/\zeta\omega_{\mathrm{n}}$ as an upper bound for the settling time.

6. It is found that a linear system (not necessarily an oscillator) has the impulse response $g(t) = e^{-2t} - e^{-4t}$. What will be the step response of the system? Is this a first-order system or a second-order system? Explain briefly. Can $g(t)$ be the impulse response of some oscillator? If so, then find the mass, damping coefficient and stiffness of such an oscillator.

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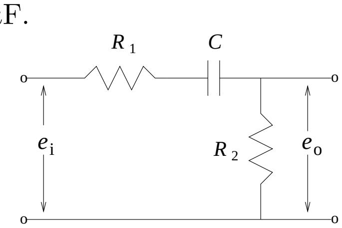
Tutorial 3, Autumn Semester, 2006

Steady-State Harmonic Response

1. A viscously damped oscillator has a spring of stiffness 525 N/m. In response to certain initial conditions, the period of vibration is found to be 1.8 s, while the amplitude is found to decay by a factor of 4.2 over a period. Determine the steady state response of the oscillator when a force $f(t) = \sin^3 \omega_n t$ acts on the mass.

2. The circuit shown below is driven by a source having zero output impedance (that is, the input voltage e_i does not depend on the current drawn by the circuit). The circuit itself is not loaded at its output. If $e_i(t) = \cos^2(10t)$, then find the output voltage e_o in the steady state. Take $R_1 = R_2 = 10K\Omega$ and $C = 1\mu$F.



3. When a viscously damped oscillator is excited by a sinusoidal force of 1N amplitude at its natural frequency, the steady state displacement is found to have an amplitude of 0.58 cm. When the excitation frequency and the amplitude are decreased by a 20%, the steady state displacement amplitude is found to be 0.37cm. Find the damping ratio ζ and the stiffness of the oscillator.

4. A vibration pickup is modelled by a spring, mass and a dashpot mounted on a moving platform. The output of the vibration pickup is proportional to the extension in the spring. Find the transfer function from the displacement of the base and the output of the pickup. A vibration pickup having a damping ratio of 0.25 and an undamped natural frequency of 1 Hz is used to measure the displacement of a structure vibrating at 4 Hz. If the pickup indicates a displacement amplitude of 0.052 cm, what is the correct amplitude? What is the frequency range over which the pickup can measure displacement amplitudes with an accuracy of within 1 %?

5. A vehicle of mass 1200 kg moves over a rough road whose surface varies sinusoidally with an amplitude 0.05 m and wavelength 6 m. Find the displacement amplitude of the vehicle when the vehicle travels at 100 km/hr. Assume that the vehicle suspension may be modelled by a spring of stiffness 400 kN/m in parallel with a damper with damping ratio $\zeta = 0.5$. How would the displacement amplitude change if the vehicle were to travel at 80 km/hr instead?