System Modeling, Dynamics and Control

Tutorial 4, Autumn Semester, 2006

Modeling and Block Diagram Reduction

1. This problem concerns an industrial temperature control system designed to regulate the temperature θ of materials being mixed in a tank. The temperature in the tank is measured by means of a thermocouple. The thermocouple output is amplified to produce a voltage $b = k_b\theta$ that is proportional to the temperature. The desired temperature is set by means of a suitable callibrated voltage r. The error e between the voltages r and b is amplified by an amplifier of gain K_a to produce a voltage e_1 that energizes the coil of a solenoid valve. The coil has resistance R and inductance L, and produces a force $F = K_s i_s$ proportional to the current i_s in the coil. This force acts on the armature of the valve having mass M, damping coefficient C and stiffness K. The displacement x of the solenoid armature controls the flow rate of steam that is used to heat the contents of the tank. The flow rate $q = K_q x$ of the steam is directly proportional to the displacement x of the solenoid armature. The transfer function between the tank temperature and the steam flow rate q is a first-order transfer function having time constant τ and a DC gain of M. Draw a (labelled) block diagram depicting all the components and quantities mentioned above and showing the transfer function of each block. Using block diagram reduction, find the closed-loop transfer function between the voltage r and the tank temperature θ. Write down a state space model for the closed-loop system using suitable state variables.

2. An insulated tank of constant cross section is supplied with liquid at controlled but variable volume inflow rate V_i and temperature Θ_i. The liquid leaves the tank through a fixed valve after being heated by a heater.

   (a) If the height of the liquid in the tank is H, the outflow temperature and volume flow rate are Θ_o and V_o, respectively, and the heater heat input rate is Q_i, then show that

   $$AH\dot{\Theta}_\mathrm{o} = V_\mathrm{i}(\Theta_\mathrm{i} - \Theta_\mathrm{o}) + (\rho c)^{-1}Q_\mathrm{i},$$

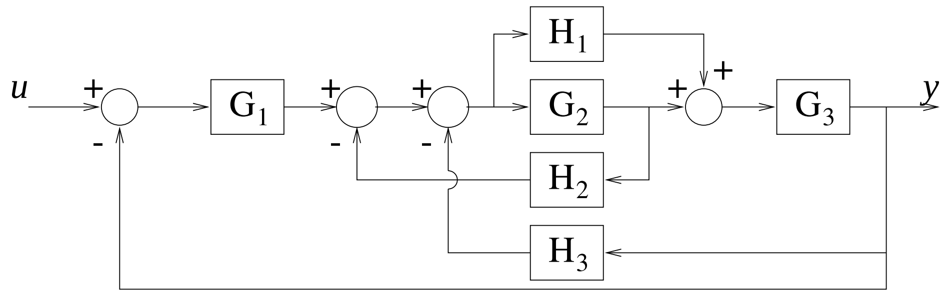
   where A is the cross sectional area of the tank, and ρ and c are the constant density and specific heat capacity, respectively, of the liquid. Also give equations governing the variations in the height and the volume outflow rate. State any assumptions you make explicitly.

   (b) Write down equations relating equilibrium values of the quantities Θ_i, Θ_o, V_i, V_o, H and Q_i.

   (c) Write down a linearized model relating perturbations in the quantities Θ_i, Θ_o, V_i, V_o, H and Q_i from their equilibrium values.

   (d) Find the response of the perturbation in the outflow temperature to a small sudden increase in the volume inflow rate from its equilibrium value.

3. Simplify the block diagram given below and find the transfer function between the input u and the output y. Show each step of simplification.



System Modeling, Dynamics and Control

Tutorial 5, Autumn Semester, 2006

Steady State Analysis

1. Let $E(s)$ denote the Laplace transform of a signal e. Under what conditions on E does $\int_0^\infty e(t)dt$ exist? What is the value of the integral when it exists?

2. In each of the following cases, determine if $\lim_{t\to\infty} e(t)$ exists, and find it if it does.
(a) $E(s) = 10(s+1)^{-2}(s^2+2s+2)^{-1}$ (b) $E(s) = 13s^{-1}(s^2+4s+13)^{-1}$.

3. Design a PI controller for the system $G(s) = (s-2)^{-1}$ such that the closed-loop system responds with zero steady state error to step inputs.

4. A unity feedback system has the transfer function $G(s) = 14(s+3)s^{-1}(s+5)^{-1}$ in the foward path. For which of the three inputs, step, ramp and parabolic, will the closed-loop system exhibit perfect tracking in the steady state? Find the static position, velocity and acceleration constants of the closed-loop system.