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Prob 66:

The direction of Y axis can be found from the direction of x and z axis (which are given).
The angular velocity would point along negative Y axis. thus it would have body components $(0, -\Omega, 0)$.
Ω: magnitude of angular velocity of satellite around earth.
The Y axis should be major principal axis as orbital angular velocity is in that direction and hence that is the most stable axis. Taking any other axis as the major principal axis would not result in a stable configuration.
Let the angular velocity imparted by the ejected gas be ω_1 along X axis and ω_3 along Z axis.
(since the thruster is located on the Y axis, no angular velocity would be imparted in that direction).

Thus, $\dot{\omega}_2 = 0 \quad \therefore \omega_2 = \Omega$

From Euler's equations,
($\because$ no external moment is applied).

$$\dot{\omega}_1 = \left(\frac{I_2 - I_3}{I_1}\right) \omega_2 \times \omega_3$$

Given: $I_2 = 2 \times I_3 = 2 \times I_1$

$$\therefore \dot{\omega}_1 = \frac{(2I_3 - I_3)}{I_3} \omega_2 \times \omega_3 = \Omega \omega_3 .$$

$$\dot{\omega}_3 = \frac{I_1 - I_2}{I_3} \omega_1 \omega_2 = \frac{-I_3}{I_3} \omega_1 \omega_2 = -\omega_1 \omega_2 = -\omega_1 \Omega .$$

$$\ddot{\omega}_1 = \Omega \dot{\omega}_3 = -\Omega^2 \omega_1$$

$\therefore$ $\omega_1 = A \sin(\Omega t) + B \cos(\Omega t)$

~~$\omega_3 = -\omega_1 \Omega$~~

~~$= -\Omega (A \sin(\Omega t) + B \cos(\Omega t))$.~~

Similarly,

$$\ddot{\omega}_3 = -\Omega^2 \omega_3$$

$\therefore$ $\omega_3 = ~~A~~\ C \sin(\Omega t) + D \cos(\Omega t)$

Now, T : time period for rotation $= \dfrac{2\pi}{\Omega}$

Hence ω_1 at $t_1 = t + T$ is

$$\omega_1 = A \sin(\Omega(t+T)) + B \cos(\Omega(t+T))$$

$$= A \sin\left(\Omega\left(t + \frac{2\pi}{\Omega}\right)\right) + B \cos\left(\Omega\left(t + \frac{2\pi}{\Omega}\right)\right)$$

$$= A \sin(\Omega t + 2\pi) + B \cos(\Omega t + 2\pi)$$

$$= \omega_1 \text{ (at time } t)$$

Similarly for ω_3.

Thus, body components of angular velocity repeat after every orbit.