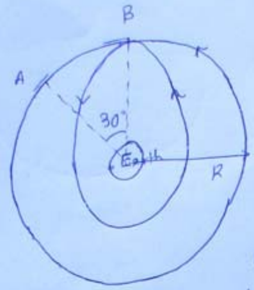
24] Let the radius of the circular orbit by R instead of 'a' so that we can use 'a' in the conventional way for the semimajor axis of an ellipse.



One way that the supply ship could intercept the space station using one impulsive thruster firing is as follows:-

Let at some time, the station be at pt. A and the supply ship at pt. B as shown in the figure. We wish to put the supply ship in an elliptical orbit with the apogee at B so that the time period of this orbit would be the time required by the space station to reach from pt. A to B on the circular orbit.

Time taken by station to reach B from A $= \dfrac{(2\pi - \pi/6)R}{\sqrt{\frac{u}{R}}}$

$\therefore$ Time period of elliptical orbit, $T = \dfrac{11\pi}{6}\sqrt{\dfrac{R^3}{u}}$ ...(1)

Time period of elliptical orbit of semimajor axis, a,

$$T = 2\pi\sqrt{\frac{a^3}{u}} \quad (2)$$

$\therefore$ from (1) & (2),

$$2\pi\sqrt{\frac{a^3}{u}} = \frac{11\pi}{6}\sqrt{\frac{R^3}{u}}$$

$$\Rightarrow a^3 = (11/12)^2 R^3$$

$$\Rightarrow \quad a = \left(\frac{11}{12}\right)^{2/3} R \quad \ldots (3)$$

For an elliptical orbit,

$$r_a = \frac{H^2/\mu}{1-e}$$

$$\Rightarrow \quad R = \frac{H^2/\mu}{1-e} \quad \ldots (4)$$

Also, $\quad a = \frac{H^2/\mu}{1-e^2} \quad \ldots (5)$

from (4) & (5),

$$R(1-e) = a(1-e^2)$$

$$\therefore \; R = a(1+e) \quad \ldots (6).$$

from (3),

$$R = \left(\frac{11}{12}\right)^{2/3} R(1+e)$$

$$\therefore \; 1+e = \left(\frac{12}{11}\right)^{2/3}$$

$$\therefore \; e = (12/11)^{2/3} - 1 \quad \ldots (7).$$

Also, velocity at ~~perigee~~ apogee is given by,

$$v_a = \sqrt{\frac{\mu}{a}\left(\frac{1\mp e}{1+e}\right)}$$

$$\therefore \; v_a = \sqrt{\frac{\mu}{\left(\frac{11}{12}\right)^{2/3} R} \left(\frac{1+\left(\frac{12}{11}\right)^{2/3}-1}{1-\left(\frac{12}{11}\right)^{2/3}+1}\right)^{-1}}$$

$$\cancel{\therefore \; v_p = \sqrt{\frac{\mu}{R}} \left[\left(\frac{12}{11}\right)^{2/3} \left(\frac{12}{11}\right)^{2/3} \frac{1}{\left(2-(12/11)^{2/3}\right)}\right]^{1/2}}$$

$$\therefore \; v_a = \sqrt{\frac{\mu}{R}} \left[\left(\frac{12}{11}\right)^{2/3} \frac{\left(2-(12/11)^{2/3}\right)}{(12/11)^{2/3}}\right]^{1/2}$$

$$\therefore \; v_a = 0.969 \sqrt{\mu/R} \quad \ldots (8)$$

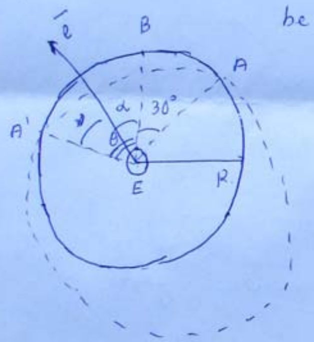
∴ Velocity change to be imparted to the space supply ship at point B, (3)

$\Delta V = V_a - V_c$ where $V_c \rightarrow$ velocity along the circular orbit

$$\therefore \Delta V = \sqrt{\frac{\mu}{R}}\left[0.969 - 1\right]$$

$$\therefore \Delta V = -0.031\sqrt{\frac{\mu}{R}} \quad \ldots \underline{\underline{ANS}}$$

In general, if we want the supply ship to intercept the ~~supply~~ space station after the space station has travelled an angular displacement θ from its current position, the ~~spa~~ supply ship would have to be put in an elliptical orbit with its eccentricity vector $\bar{e}$ ~~wo~~ making an angle α with EB. B is the initial position of the station.



By symmetry,

$$30 + \alpha = \theta - \alpha$$

$$\Rightarrow \alpha = \left(\frac{\theta - 30}{2}\right)^{\circ} \ldots (1)$$

Also, $\nu = (\theta - \alpha) \ldots (2)$

Time required by the space station to travel θ:

$$T_\theta = \frac{\theta\pi}{180}\sqrt{\frac{R^3}{\mu}} \ldots (3).$$

∴ for the supply ship, $(t - t_p) = T_\theta/_2 \ldots (4)$

~~for~~

for the supply ship,

$$\tan E/_2 = \sqrt{\frac{1-e}{1+e}}\tan \nu/_2 \quad ..(5)$$

$$E - e\sin E = \sqrt{\frac{\mu}{a^3}}(t-t_p) \quad ..(6)$$

Eliminating E from eqns (5) & (6), we obtain, some function, $f(e, a, \theta) = 0 \;.(7)$

By the parametric eqn. of the elliptical orbit,

$$R = \frac{H^2/\mu}{1+e\cos\nu} \quad ..(8)$$

$$\Rightarrow \quad e = \phi(H, \theta) \;.(9)$$

Also,

$$H = \sqrt{\mu a(1-e^2)}$$

$$\therefore a = \gamma(H) \quad ..(10)$$

$\therefore$ from (7), (9), (10),

some function, $g(H, \theta) = 0 \quad .(11)$

solving (11) for H as θ is known, gives us the required orbit.

Nice idea. But this will require 2 impulses.

SPB

~~Classical Dy. AE 459~~ / Question 25
Space Flight Mechanics AE 415
Submitted by Jamie Gawarne / 04d01022.

In the perifocal frame, the $\hat{p}$ is along the eccentricity. $\hat{q}$ is along the semi latus rectum, $\bar{r}$ and $\bar{v}$ are given by

$$\bar{r} = r[\cos\nu\,\hat{p} + \sin\nu\,\hat{q}] \quad \text{and} \quad \dot{\bar{r}} = \sqrt{\frac{\mu}{a(1-e^2)}}\,[-\sin\nu\,\hat{p} + (e+\cos\nu)\,\hat{q}]$$

According to the question, the satellite is at semi-latus rectum and semimajor axis is rotated by 90°.

The initial orbit has semimajor axis a and eccentricity e. The final orbit has semimajor axis a' and eccentricity e'

semilatus rectum of initial orbit $= \frac{h^2}{\mu} = a(1-e^2)$

equals perigee of the final orbit $a'(1-e)$

$$\Rightarrow \quad a(1-e^2) = a'(1-e)$$

$$a' = a(1+e)$$

Velocity at the semilatus rectum of initial orbit

$$\bar{V}_i = \sqrt{\frac{\mu}{a(1-e^2)}}\left[-\hat{p} + e\hat{q}\right] \Rightarrow |\bar{V}| = \sqrt{\frac{\mu}{a(1-e^2)}} \times \sqrt{1+e^2}$$

velocity at the perigee of final orbit.

$$|\bar{V}_0| = \sqrt{\frac{\mu(1+e)}{a'(1-e)}} \Rightarrow \sqrt{\frac{\mu(1+e)}{a(1-e^2)}} = \sqrt{\frac{\mu}{a(1-e)}}$$

impulse required $= |\bar{V}_0| - |\bar{V}_i|$ → WRONG.

these velocities are along different directions.

$$\Rightarrow \sqrt{\frac{\mu(1+e)}{a(1-e^2)}} - \sqrt{\frac{\mu(1+e^2)}{a(1-e^2)}}$$

$$\Rightarrow \sqrt{\frac{\mu}{a(1-e^2)}}\left[\sqrt{1+e} - \sqrt{1+e^2}\right]$$

$$\bar{V}_i = \sqrt{\frac{\mu}{a(1-e^2)}}\left[-\hat{p} + e\hat{q}\right].$$

In the new orbit, velocity has to be along $-\hat{p}$ since the point becomes the perigee of the orbit.

$$\therefore \bar{V}_f = -\sqrt{\frac{\mu}{a(1-e)}}\,\hat{p}.$$

$\therefore$ Impulse required is $\bar{V}_f - \bar{V}_i$.

SBR

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PROBLEM 29

1 year = 365.25 mean solar days

1 mean solar day = 24 hrs.

As the photographs of any given location are required to be taken under the same lighting conditions, the satellite has to be sunsynchronous.

For a sun-synchronous orbit, the nodal regression rate is 360°/year.

$$\Rightarrow \frac{d\Omega}{dt} = \frac{2\pi}{365.25 \times 24 \times 3600} = 1.991 \times 10^{-7} \text{ rad/sec.}$$

As for a circular orbit, $e = 0$

$$\Rightarrow \text{Regression rate, } \frac{d\Omega}{dt} = -\frac{3}{2} n J_2 \cos i \left(\frac{Re}{a}\right)^2 \ldots\ldots (1).$$

It is required that the ground trace should repeat after every N orbits. While, the satellite completes its N orbits, the earth rotates M times.

If the effects due to the rotation of the orbital plane are neglected and also the effects due to the revolution of earth are neglected

$$M \text{ (one sidereal day)} = N \text{ (Time period of satellite)} \quad ..(2)$$

one sidereal day = 23 hrs 56 min 4.09 secs

~~= 88804.09 secs.~~

= 86164.09 secs.

$$T = 2\pi \sqrt{\frac{a^3}{\mu}}$$

Typically $N = 12$ for $m = 1$ (source: Wikipedia).

$$\Rightarrow \quad 12\,T = 86164.09$$

$$T = 7180.3408 \text{ secs.}$$

$$2\pi \sqrt{\frac{a^3}{\mu}} = 7180.3408 \text{ secs.}$$

$$\Rightarrow \quad a = 8044.3246 \text{ kms.}$$

Altitude of satellite $= a - R_e = 1666.3246$ kms.

$$n = \frac{2\pi}{T} = \frac{2\pi}{7180 \cdot 3408}$$

From (1):-

$$-\frac{3}{2} n J_2 \cos i \left(\frac{Re}{a}\right)^2 = 1 \cdot 991 \times 10^{-7}$$

$$J_2 = 1 \cdot 08 \times 10^{-3}$$

$$Re = 6378 \text{ km}$$

Inclination, $i = 102 \cdot 91^\circ$.

$\Rightarrow$ the orbit is retrograde.

Typically, altitude of a sun-synchronous orbit is from 600 to 800 kms. and the inclination is around 98°.

Sun-synchronous designed to fly over the US, has i of 132° or less and altitude of 4600 km or less.

(Source: Wikipedia)

Problem 32: Outline of solution by S. P. Bhat (I98002).

Neglecting nodal regression, the given information implies that the satellite completes 16 orbits each time the earth completes one rotation about its axis.

Hence Orbital period = $\dfrac{1 \text{ sidereal day}}{16}$.

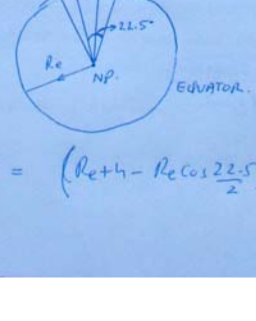
Use this to find a.

Since the orbit is circular, $e = 0$

Hence find inclination i by setting nodal regression rate $\dot{\Omega} = -\frac{3}{2} n \frac{J_2 \cos i}{(1-e^2)^2} \left(\frac{R_e}{a}\right)^2$ equal to

$$2\pi / 1 \text{ solar year } (= 365.25 \text{ days}).$$

The satellite crosses the equator at the same 16 points. Hence, for full coverage, at each crossing, it must be able to see an area which covers 1/16th of the equator, that is, $360/16 = 22.5°$ of the equator. To find the field of view β, use the following figure.



h - altitude

β - field of view.

$$R_e \sin \frac{22.5}{2} = \left(R_e + h - R_e \cos \frac{22.5}{2}\right) \tan \frac{\beta}{2}.$$