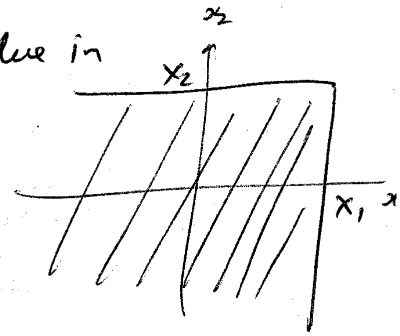
A random vector is described by the joint PDF of its elements. Thus

$F_x(X)$ (also written as $F_{x_1,x_2}(X_1,X_2)$)

$= \Pr.\,(x_1 \le X_1 \;\&\; x_2 \le X_2)$. $\quad x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$, $X=\begin{bmatrix}X_1\\X_2\end{bmatrix}$.

$F_x(X)$ is the probability that x takes its value in the shaded set in the figure.



The ~~joi~~ pdf of f, that is, the joint pdf of its elements (also denoted $f_{x_1,x_2}(X_1,X_2)$) is the ~~fast~~

$f_x(X)$ s.t

$$\Pr.\,(x\in A) = \iint_A f_{x_1x_2}(X_1,X_2)\,dX_1\,dX_2 .$$

$\Pr.\,(x\in A) =$ area ~~be~~ above A & below the graph of f_x.

Thus $$F_{x_1x_2}(X) = \int_{-\infty}^{X_1}\int_{-\infty}^{X_2} f_x(X)\,dX_1\,dX_2 .$$

$$\therefore\; f_{x_1x_2}(X) = \frac{\partial^2 F_{xy}}{\partial X_1\,\partial X_2}(X_1,X_2).$$

We will find it simpler to treat x_1, x_2 as the elements of a random vector $x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$, & treat the joint PDF & pdf as fns. of a vector $X=\begin{bmatrix}X_1\\X_2\end{bmatrix}$.

The mean of x_i, $i=1,2$ is defined by

$$\bar{x}_i \ (\text{or } E(x_i)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_i f_x(x)\, dx_1 dx_2 ,$$

The variance of x_i is $\sigma_{x_i}^2 = E[(x_i - \bar{x}_i)^2]$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x_i - \bar{x}_i)^2 f_x(x)\, dx_1 dx_2$$

The variance between x_1 & x_2 is

$$\sigma_{x_1 x_2} = E\left((x_1 - \bar{x}_1)(x_2 - \bar{x}_2)\right).$$

x_1 & x_2 are uncorrelated if $\sigma_{x_1 x_2} = 0$

Note that $\sigma_{x_1 x_2} = \sigma_{x_2 x_1}$.

The covariance matrix of the random vector x is

$$P_{xx} = E\left[(x-\bar{x})(x-\bar{x})^T\right] = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1 x_2} \\ \sigma_{x_2 x_1} & \sigma_{x_2}^2 \end{bmatrix}$$ is symmetric & pos. definite.

P_{xx} is diagonal if x_1 & x_2 are uncorrelated.

The multivariable Gaussian pdf is

$$f_x(x) = \frac{1}{2\pi\sqrt{\det P}}\, e^{-\frac{1}{2}(x-\mu)^T P^{-1}(x-\mu)}$$

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \quad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$$ - symm. +ve definite