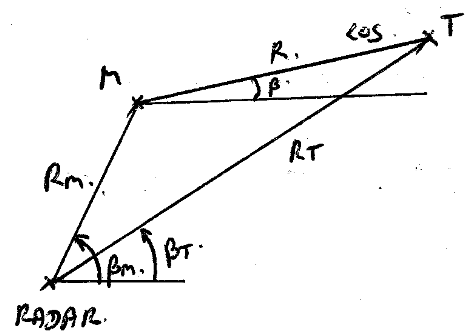
Types of command guidance

1) Beam rider guidance: A beam is continuously pointed at the target, either from a fixed station, or an aircraft. The missile guidance system attempts to ride along the beam till intercept. The guidance problem is to reduce the deviation of the missile from the beam.

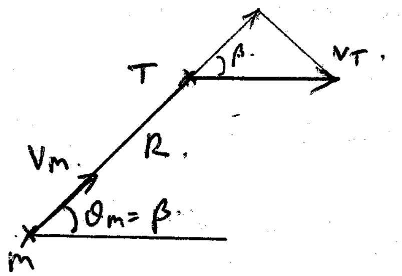
2) Proportional command guidance: The missile & target are tracked by a radar which yields the range & LOS direction to each. These measurements are processed to yield β & $\dot{\beta}$ estimates, & generate a guidance command (using proportional guidance law) which is uplinked to the missile.



Pursuit guidance: Pursuit guidance requires the missile velocity vector to be always pointed towards the target. Hence, the missile velocity vector is along the LOS, that is,

$$\theta_M = \beta.$$

For simplicity, we assume that the target moves in a straight line at a constant speed, that is the target is not maneuvering. Without loss of generality, we may assume that $\theta_T = 0$. Hence the engagement geometry is as shown below.



range rate $\dot{R} = V_T \cos\beta - V_m$,

LOS rate $\dot{\beta} = -\dfrac{V_T \sin\beta}{R}$.

Note: $\dot{\beta} = 0$ if & only if $\beta = 0$ (tail chase)

or $\beta = \pi$ (head on collision).

This implies that the LOS ~~rate~~ always rotates unless $\beta = 0$ or π,

Since $\theta_m = \beta$, it follows that the missile always turns. (unless $\beta = 0$ or π).

We have $\dfrac{dR}{d\beta} = \dfrac{\dot{R}}{\dot{\beta}} = (-\cot\beta + r\,\mathrm{cosec}\,\beta)R$, where $r = \dfrac{V_m}{V_T}$

$\therefore \dfrac{dR}{R} = (-\cot\beta + r\,\mathrm{cosec}\,\beta)\,d\beta$.

$\therefore \ln R = -\ln|\sin\beta| + r\ln|\tan\beta/2| + \text{const.}$

$$\therefore \quad \frac{R|\sin\beta|}{|\tan\beta/2|^r} = K = \frac{R_0|\sin\beta_0|}{|\tan\beta_0/2|^r}$$

$$\therefore \quad R = \frac{K\,|\tan\beta/2|^r}{|\sin\beta|}.$$

Note that, as intercept draws closer, $R \to 0$. Since $|\sin\beta|$ is bounded, it ~~must~~ follows that $\beta \to 0$ as intercept nears.. ~~Ee~~ $\therefore$ β is small in the terminal phase. i.e.

In the terminal phase, $R = K\left(\frac{|\tan\beta/2|^r}{|\sin\beta|}\right) \sim \frac{K\beta^r}{2^r\beta} = \frac{K\beta^{r-1}}{2^r}$

$\therefore$ for R to approach zero as $\beta \to 0$, we need $r > 1$. Pursuit guidance works only if the missile has a speed advantage over the target.

The LATAX required is

$$\text{LATAX} = \dot{\theta}_M V_M = \dot{\beta} V_M = -\frac{V_T V_M \sin\beta}{R}$$

$$= -\frac{V_T V_M \sin\beta\,|\sin\beta|}{K\,|\tan\beta/2|^r}$$

In the terminal phase, β is small $\therefore$

$$\text{LATAX} \sim -\frac{V_T V_M 2^r \beta^{2-r}}{K}$$

$\therefore$ For bounded LATAX requirement, need $r < 2$.

Since R reaches zero when β reaches zero, the time-to-intercept is the time required for β to reach zero.

Now $$\dot{\beta} = \frac{-V_T \sin\beta}{R} = -\frac{V_T \sin\beta |\sin\beta|}{k|\tan\beta/2|^r}$$

$$\therefore -d\beta \cdot \frac{k|\tan\beta/2|^r}{V_T \sin\beta |\sin\beta|} = dt.$$

$$\therefore t_f - t_0 = \int_0^{\beta_0} \frac{k|\tan\beta/2|^r}{V_T \sin\beta|\sin\beta|} d\beta.$$

The pursuit ~~analysis~~ guidance problem was originally formulated ~~for the~~ & solved by Pierre Bouguer, ~~(1698-1758)~~ a French mathematician, in 1732, for a pirate ship chasing a merchant ship sailing along a straight line, & provides a model for pursuit by animals & birds of prey.