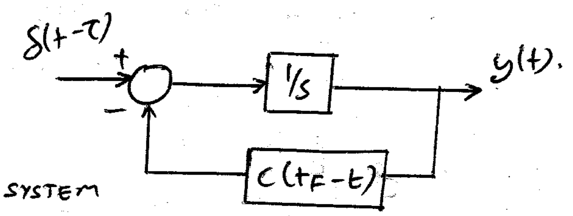
In order to understand the relationship between solutions of the original system and the adjoint system, Consider the time varying system

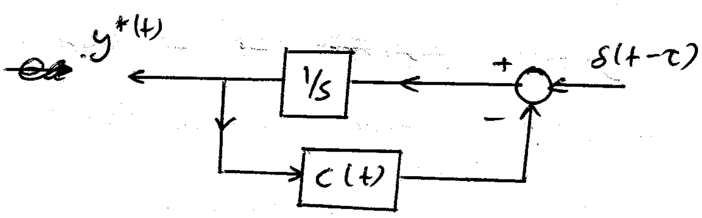
$$\dot{y}(t) + c(t_F - t)y(t) \quad \delta(t-\tau), \quad \text{where } \tau \le t \le t_F.$$

This system may be represented by the block diagram



ORIGINAL SYSTEM

The block diagram of the adjoint system is



ADJOINT SYSTEM

The ODE of the adjoint system is

$$\dot{y}^+(t) + c(t)\,y^+(t) = \delta(t-\tau).$$

To solve the above equation, multiply both sides by the integrating factor $e^{\int_0^t c(s)ds}$ to get

$$e^{\int_0^t c(s)ds}\,\dot{y}^+(t) + c(t)\,e^{\int_0^t c(s)ds}\,y(t) = e^{\int_0^t c(s)ds}\,\delta(t-\tau).$$

$$\therefore \frac{d}{dt}\left[e^{\int_0^t c(s)ds}\,y^+(t)\right] = e^{\int_0^t c(s)ds}\,\delta(t-\tau).$$

$$\therefore e^{\int_0^t c(s)ds}\,y^+(t) - y(0) = \int_0^t e^{\int_0^h c(s)ds}\,\delta(h-\tau)\,dh$$

Assuming the initial condition to be zero and recalling that

$$\int_0^t f(h)\,\delta(h-\tau)\,dh = f(\tau), \quad \text{we get}$$

$$y^*(t) = e^{-\int_0^t c(s)\,ds}\; e^{\int_0^\tau c(s)\,ds} = e^{-\int_\tau^t c(s)\,ds}.$$

$y^*(t)$ is the response of the adjoint system to an impulse input applied at τ. We denote this impulse response by $w^*(t, \tau)$.

Hence $$w^*(t, \tau) = e^{-\int_\tau^t c(s)\,ds}.$$

Since the original system ~~has the~~ is similar in form, we can easily write its impulse response function as

$$w(t, \tau) = e^{-\int_\tau^t c(t_F - s)\,ds}$$

The change of variables $t_F - s = h$ yields

$$w(t, \tau) = e^{-\int_{t_F - t}^{t_F - \tau} c(h)\,dh}$$

A comparison of w & w^* shows that

$$\boxed{w^*(t, \tau) = w(t_F - \tau,\, t_F - t).}$$

It turns out that the above relationship holds between the impulse response functions of any linear system and its adjoint.

In the case of linearized miss distance analysis, we are interested in $\omega(t_F, 0)$, the output of the homing loop ~~in~~ at $t = t_F$ in response to an impulse applied at $\tau = 0$. Putting $t = t_F$ & $\tau = 0$ gives

$$\omega(t_F, 0) = \omega^*(t, \tau) = \omega^*(t_F, 0).$$

Thus the required miss distance at $t = t_F$ is the response of the adjoint ~~system~~ of the homing loop at $t = t_F$. The advantage is that the adjoint does not contain the terminal time t_F, & hence a single simulation yields the miss distance as a function of the flight time t_F.

<u>Miss distance due to FCS lags.</u> (Zarchan p. 43)

We will use the method of adjoints to obtain a closed-form expression for the miss distance as a function of flight time ~~in the case wher~~ resulting from heading error & step target maneuver in the case where the FCS has a lag.

For this, we draw the homing loop with the FCS lag, & perform ~~block d~~ permissible block diagram manipulations, construct the adjoint, and then analyze the adjoint.