Lambert's problem:

Given $t_0, r_0, \theta_0\ (=0)$, & $t_F, r_F\ (=R_e), \theta_F$,

find V_0, Γ_0.

The hit equation yields ~~several~~ a family of trajectories parametrized by Γ_0 and having different flight times.

$$V_0 = V_0(r_0, \Gamma_0, \theta_f, r_f) \qquad \text{hit eqn.}$$

$$t_F = t_F(r_0, V_0, \Gamma_0, \theta_f) \qquad \text{flight time.}$$

Substituting for V_0 in the flight time equation from the hit equation yields the flight time as a function of Γ_0 under the constraint that an intercept is achieved.

Thus $\quad t_F = f(\Gamma_0)$. $\quad (\star)$

The solution to Lambert's problem involves solving $(\star)$ numerically for Γ_0, & then using the hit equation to find V_0.

The solution of $(\star)$ can be achieved by any of the following techniques.

1) Bisection method. (does not require f', slowest to converge)
2) Secant method. (does not require f', relatively faster)
3) Iterations. (may not converge)
4) Newton's method. (fast, requires f').

Lambert Guidance (or velocity-to-be-gained guidance).

The objective of the Lambert guidance strategy is to drive the missile to a position & velocity which solve the Lambert problem. This position & velocity serve as the cut off condition. The guidance strategy involves computing the velocity required at the current position to solve the Lambert problem, & then applying an acceleration along the difference between this required velocity vector & the current velocity vector.

Thus, if $v_{Lambert}(r_0, \theta, t)$ is the ~~initial~~ velocity required at the position (r, θ) at time t to intercept the target at time t_F, & $v_m(t)$ is the velocity of the missile at time t, then the acceleration command is

$$a_c(t) = M \frac{v_{Lambert}(r(t), \theta(t), t) - v_m(t)}{\| v_{Lambert}(r(t), \theta(t), t) - v_m(t) \|}$$

where M is the magnitude of the ~~boost~~ thrust acceleration.

The thrust is cut off when $v_m(t) = v_{Lambert}$.

At each guidance time step, $v_{Lambert}$ is calculated using an iterative numerical procedure.

~~Miss an~~ Linearized miss analysis:

For any given cutoff condition, the location of the impact point is determined by an implicit equation of the form

$$H(r_0, \cancel{\theta_0}^{=0}, V_0, \Gamma_0, \theta_F) = 0, \quad \text{where}$$

$$H(r_0, \theta_0, V_0, \Gamma_0, \theta_F)$$

$$= R_e\left(1 - \frac{h^2}{\mu r_0}\tan\Gamma_0 \sin(\theta_F - \theta_0) + \left\{\frac{h^2}{\mu r_0} - 1\right\}\cos(\theta_F - \theta_0)\right) - \frac{h^2}{\mu}.$$

The presence of perturbations in the cutoff conditions will lead to a perturbation in the angular location of the impact point.

If ~~the~~ perturbed cutoff conditions

$$r_0 + \delta r_0, \quad \theta_0 + \delta\theta_0, \quad V_0 + \delta V_0, \quad \Gamma_0 + \delta\Gamma_0,$$

result in the perturbed impact point $\theta_F + \delta\theta_F$, then

$$H(r_0 + \delta r_0, \theta_0 + \delta\theta_0, V_0 + \delta V_0, \Gamma_0 + \delta\Gamma_0, \theta_F + \delta\theta_F) = 0.$$

To estimate $\delta\theta_F$ using linearization, we consider only the 1st order terms in the above equation. Hence

$$\left.\frac{\partial H}{\partial r_0}\right|_{\text{nominal}} \delta r_0 + \left.\frac{\partial H}{\partial \theta_0}\right|_{\text{nom}} \delta\theta_0 + \cdots + \left.\frac{\partial H}{\partial \theta_F}\right|_{\text{nom.}} \delta\theta_F = 0$$

$$\therefore \text{miss} = R_e\,\delta\theta_F = -R_e\left[\left.\frac{\partial H/\partial r_0}{\partial H/\partial \theta_F}\right|_{\text{nom.}} \delta r_0 + \cdots + \left.\frac{\partial H/\partial r_0}{\partial H/\partial \theta_F}\right|_{\text{nom.}}\right].$$

Thus $\cancel{\delta\theta_F}$ miss M $= M_{r_0}\,\delta r_0 + M_{\theta_0}\,\delta\theta_0 + M_{v_0}\,\delta v_0 + M_{\gamma_0}\,\delta\gamma_0$.

The coefficients M_{r_0}, M_{θ_0} ... are called miss coefficients.

Letting $\delta x = [\delta r_0, \ \delta\theta_0 . \ \delta v_0, \ \delta\gamma_0]^T$

& $M_x = [M_{r_0} \ \ M_{\theta_0} \ .. \ M_{v_0} \ M_{\gamma_0}]^T$,

we have $\cancel{\delta\theta_F}$ $M = M_x^T \delta x$.

If the perturbation vector δx in the cutoff condition has a covariance matrix P_x, then the miss will have a variance

$$\sigma_{\theta_F}^2 = M_x^T P_x M_x .$$

The perturbation ~~in~~ δx is due to dispersions in the cutoff condition caused by various factors operating in the boost phase (eg. winds, launch errors, off nominal propulsion performance, guidance errors).