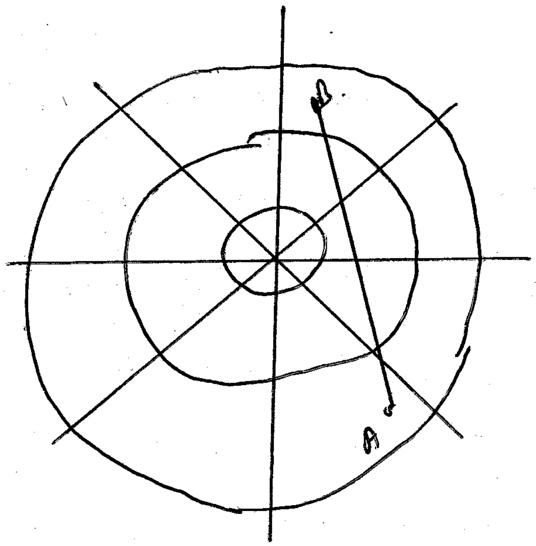
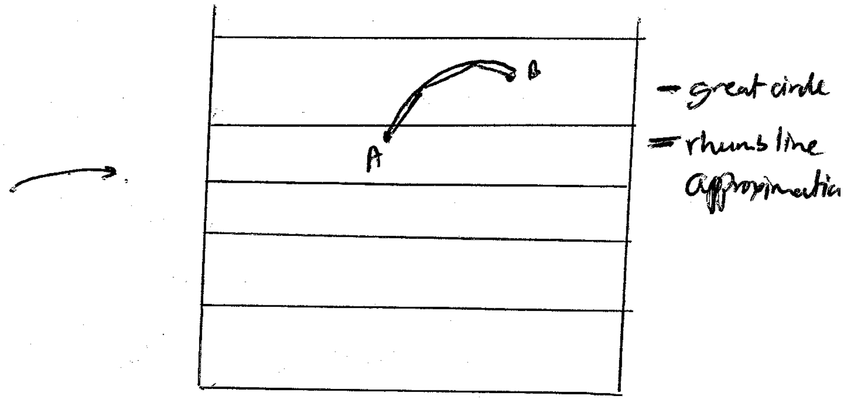
~~A rhumb line~~ The shortest route between two points on a sphere is along a great circle arc. However, a great circle is difficult to navigate along, because the course changes constantly along a great circle. On the other hand, ~~a~~ the ~~th~~ distance along a rhumbline may significantly exceed the great circle distance. ~~He~~ One alternative is to select intermediate points along the great circle route & navigate between successive ones using rhumb lines.

A great circle route is difficult to plot on a Mercator chart. An alternative plotting medium is a gnomonic chart. A gnomonic chart is made by radially projecting the surface of a sphere on a tangent plane. Great circles project onto straight lines on a gnomonic chart. The center of the chart (point of tangency) is located at a place of interest. A polar gnomonic chart has its center at ~~a~~ the pole. Parallels of latitude appear as circles, while meridians appear as radial lines. A straight line segment between pts A & B on a gnomonic chart yields points along the great circle route. These points are transferred to a Mercator chart, & ~~ap~~ ~~the~~ approximated by straight line segments which yield the courses required to follow the rhumb line approximation.



Polar gnomonic chart.



Mercator chart. (From Cotter).

Equation of a great circle.

$A - (\varphi_A, \lambda_A)$, $B - (\varphi_B, \lambda_B)$, pt. $C - (\varphi, \lambda)$.

unit vectors to A & B, C (in appropriate frame).

$$r_A = \begin{bmatrix} \cos\varphi_A \cos\lambda_A \\ \cos\varphi_A \sin\lambda_A \\ \sin\varphi_A \end{bmatrix}, \quad r_B = \begin{bmatrix} \cos\varphi_B \cos\lambda_B \\ \cos\varphi_B \sin\lambda_B \\ \sin\varphi_B \end{bmatrix}, \quad r = \begin{bmatrix} \cos\varphi \cos\lambda \\ \cos\varphi \sin\lambda \\ \sin\varphi \end{bmatrix}.$$

~~The unit vector of a~~

$$r_A \times r_B = \begin{bmatrix} \cos\varphi_A \sin\varphi_B \sin\lambda_A - \cos\varphi_B \sin\varphi_A \sin\lambda_B \\ \sin\varphi_A \cos\varphi_B \cos\lambda_B - \sin\varphi_B \cos\varphi_A \cos\lambda_A \\ \cos\varphi_A \cos\varphi_B \sin(\lambda_B - \lambda_A) \end{bmatrix}$$

A, B, C lie in the same plane.

$$\therefore r^T (r_A \times r_B) = 0.$$

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$$\Rightarrow \cos\varphi_B \sin\varphi_A \cos\varphi \sin(\lambda - \lambda_B)$$
$$+ \cos\varphi_A \sin\varphi_B \cos\varphi \sin(\lambda_A - \lambda)$$
$$+ \cos\varphi_A \cos\varphi_B \sin\varphi \sin(\lambda_B - \lambda_A) = 0.$$

Hence the latitude as a function of the longitude along a great circle is given by

$$\tan\varphi = -\frac{\left[\cos\varphi_B \sin\varphi_A \sin(\lambda - \lambda_B) + \cos\varphi_A \sin\varphi_B \sin(\lambda_A - \lambda)\right]}{\cos\varphi_A \cos\varphi_B \sin(\lambda_B - \lambda_A)}.$$

The distance along the great circle route is

$$= R\cos^{-1} r_A^T r_B = R\cos^{-1}\left[\cos\varphi_A \cos\varphi_B \cos(\lambda_A - \lambda_B) + \sin\varphi_A \sin\varphi_B\right].$$

Celestial Navigation. - (TERMS- GARDNER & CREELMAN ~~MATH~~ ~~HOFFMAN~~)

- determination of position (latitude & longitude) using measurement of positions of celestial objects

~~The~~ Celestial objects such as stars appear to form a sphere, called the celestial sphere, which appears to rotate about an axis joining the celestial poles. Earth's equatorial ~~pla~~ plane cuts the celestial sphere in the equinoctial. Planes containing the celestial poles cut the sphere along celestial meridians. The earth's orbital plane intersects the celestial sphere in the ecliptic. The sun's annual apparent motion on the celestial sphere is along the ecliptic. The 1st point of Aries is one of the points of intersection of the ecliptic and the equinoctial. It is the position of the sun at the time of vernal equinox, that is when the Sun ~~ap~~ just transits from south to north.

The position of a celestial object on the celestial sphere is described in terms of

a) Declination – "latitude" on the celestial sphere measured from the equinoctial.

b) Right ascension – "longitude measured eastward from the 1st point of Aries (0 to 360°).

c) Sidereal hour angle (SHA) – angle between the celestial meridian of the first point of Aries along the equinoctial measured westwards (0 to 360°).

$$SHA = 360 - \text{right ascension}.$$

All of the above remain fixed, and are tabulated in star charts.

The position of ~~the~~ celestial objects relative to earth is described using

a) Local hour angle (LHA) – angle between the celestial meridian of the observer and the celestial meridian of the body, measured westwards from the observer.

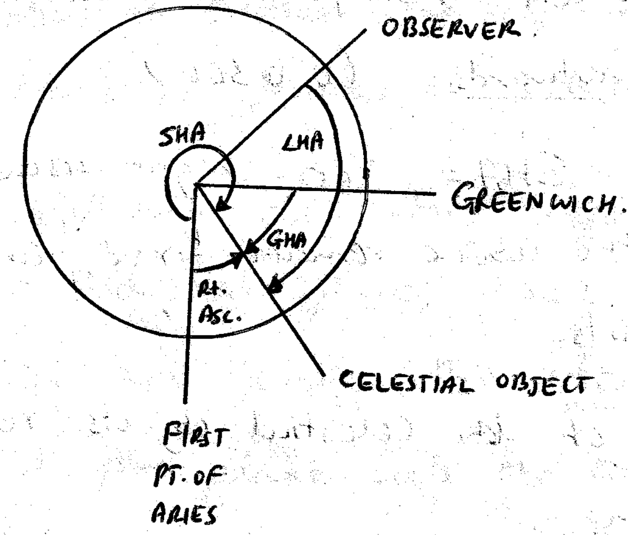
b) Greenwich hour angle (GHA) – LHA at Greenwich.

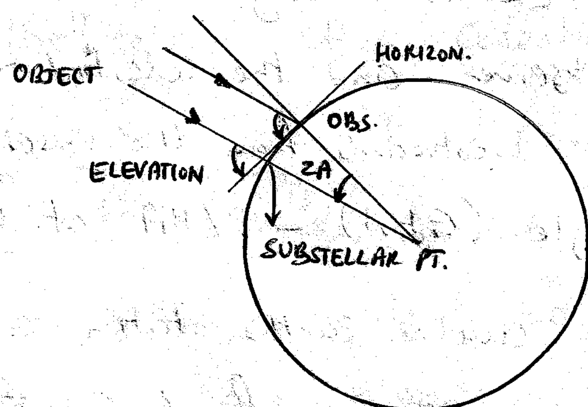
LHA & GHA change due to earth's rotation, and are listed in nautical almanacs as fn. of GMT

c) Zenith angle – arc of a vertical circle between the body and zenith. Zenith is the point on the celestial sphere directly overhead the observer. A vertical circle is a great circle passing through the zenith.

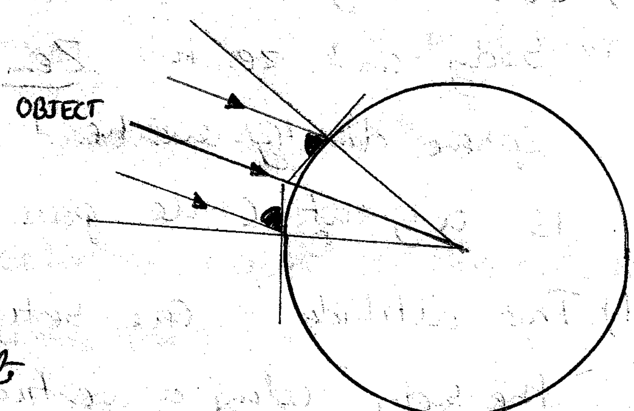
d) True altitude – arc between observers horizon and the body along a vertical circle

True altitude $= 90 - ZA$, & is also known as elevation





Each elevation measurement gives a circular LOP centered at the substellar point



Two readings are required to achieve a fix. Give two circles which intersect in generally two points