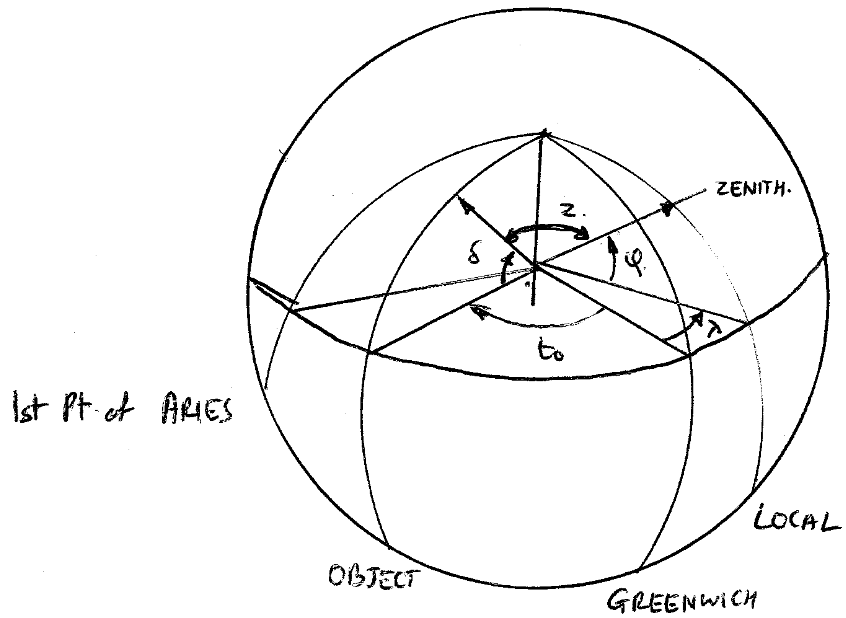

φ, λ - lat & long of observer

δ - declination of the celestial object.

z - zenith angle of the object (would also need ~~ele~~ local azimuth to locate it in the sky)

t_0 - GHA of the object

The unit vector along the zenith vector is

$$r = \begin{bmatrix} \cos\varphi \cos\lambda \\ \cos\varphi \sin\lambda \\ \sin\varphi \end{bmatrix}$$

The unit vector along the celestial object is

$$u = \begin{bmatrix} \cos\delta \cos t_0 & -\cos\delta \sin t_0 & \sin\delta \end{bmatrix}^T$$

The zenith angle is $\cos z = r^T u$.

Suppose the zenith angles to two objects are

$$r^T u_1 = \cos z_1, \qquad r^T u_2 = \cos z_2 .$$

This implies $\quad r^T(\cos z_2\, u_1 - \cos z_1\, u_2) = 0.$

Let $\quad h = \cos z_2\, u_1 - \cos z_1\, u_2 \;\;\&\;\; g = u_1 \times u_2$. & note that $g \perp h$.

Since $r^T h = 0$, $\;r$ lies in the plane formed by g & $g \times h$.

$\therefore$ Suppose $\; r = a_1 g + a_2\, g \times h.$

Since $r^T r = 1$, $\quad$ we have $\; a_1^2 g^T g + a_2^2 g^T g\, h^T h = 1.$

Also, $\;\cancel{\text{Also}}\; \cos z_1 = r^T u_1 = a_2 (g \times h)^T u_1$

$$= a_2 \left[(u_1 \times u_2) \times (\cos z_2\, u_1 - \cos z_1\, u_2) \right]^T u_1$$

$$= - a_2 \cos z_1 \,(u_2 \times u_1)^T (u_1 \times u_2)$$

$$= \cancel{-}\, a_2 \cos z_1\, g^T g.$$

$\therefore a_2 = 1/g^T g$, $\;$ which yields

$$a_1 = \pm \frac{\sqrt{g^T g - h^T h}}{g^T g}.$$

$$\therefore\; r = \frac{1}{g^T g}\left[g \times h \pm \sqrt{g^T g - h^T h}\; g \right]. \quad - \text{ position fix.}$$