AE 457/641 Navigation and Guidance

Midsemester Test: September 13, 2007, 1430 - 1630

Total Marks – 30

ONE A4 SIZE HANDWRITTEN SHEET ALLOWED. NO PHOTOCOPIES, NOTES OR PRINTED MATTER ALLOWED.

1. An observer on a moving vessel measures range rates $\dot{\rho}_{\mathrm{A}} = -4.58\mathrm{m/s}$, $\dot{\rho}_{\mathrm{B}} = -5\mathrm{m/s}$ and $\dot{\rho}_{\mathrm{C}} = -4.35\mathrm{m/s}$ to three landmarks A, B and C having coordinates $(-1, 0)$, $(0, 1.5)$ and $(1.3, 0)$, respectively. At the time of making the measurements, the position of the observer is determined to be $(0, -2.3)$. The measurement error vector $[\delta\dot{\rho}_{\mathrm{A}}\ \delta\dot{\rho}_{\mathrm{B}}\ \delta\dot{\rho}_{\mathrm{C}}]^{\mathrm{T}}$ is zero mean and has the covariance matrix
$$\begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.1 & 0.3 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}.$$

   (a) Estimate the velocity vector of the oberver using only the first two measurements (that is, A and B). Compute the covariance matrix and the RMS value of the error in the estimate obtained in this way. (4)

   (b) Find the best linear unbiased estimate of the observer's velocity vector obtained by using all the three measurements. Compute the covariance matrix and the RMS value of the error in the estimate thus obtained. (5)

2. An observer uses two laser range finders A and B to measure her distance from a landmark. The range finders A and B yield measurements of 806.63 m and 807.23 m, respectively. The standard deviations of the measurement errors of the range finders A and B are known to be 32.1cm and 33.4cm, respectively. Find the minimum variance estimate of the distance if the measurement errors of the range finders are known to be uncorrelated. Find the standard deviation of the minimum variance estimate. (4)

3. An observer measures the bearings of three landmarks A, B and C to be θ_{A}, θ_{B} and θ_{C}, respectively. The error in the bearing measurement has an unknown mean (or bias) and a standard deviation of $\sigma = 0.3°$. Hence the observer uses the differences in the bearings $\theta_{\mathrm{AB}} = \theta_{\mathrm{A}} - \theta_{\mathrm{B}}$ and $\theta_{\mathrm{BC}} = \theta_{\mathrm{B}} - \theta_{\mathrm{C}}$ to estimate his position to be (2.54,1.34).

   (a) Find the covariance matrix of the measurement error vector $[\delta\theta_{\mathrm{AB}}\ \delta\theta_{\mathrm{BC}}]^{\mathrm{T}}$. Assume that the errors in bearing measurements taken at different times (that is, $\delta\theta_{\mathrm{A}}$, $\delta\theta_{\mathrm{B}}$, $\delta\theta_{\mathrm{C}}$) are uncorrelated. (3)

(b) If the landmarks A, B and C have coordinates $(0, 2.6)$, $(0, 0)$ and $(2.26, 0)$, respectively, then find the covariance matrix of the error in the position estimate. (7)

4. The joint probability density function of two random variables x_1 and x_2 is given by

$$\begin{aligned} f_{x_1 x_2}(X_1, X_2) &= 1, \quad X_1 \in [0,1],\ X_2 \in [0,1], \\ &= 0, \quad \text{otherwise.} \end{aligned}$$

Compute the mean and the covariance matrix of the random vector $x = [x_1\ x_2]^{\mathrm{T}}$. (4)

5. Find the rhumb line distance and the great circle distance between the port of Cuddalore (11.7° N, 79.77° E) and Port Blair (almost 11.7° N, 92.5° E). (3)