Tutorial 1. 10/8/07.

1). New York: latitude $\varphi_A = 40°47' = 40.783° = 0.711$ rad. N
longitude $\lambda_A = 73°58' = 73.966° = 1.29$ rad. W

Cardiff: latitude $\varphi_B = 51°30' = 51.5° = 0.898$ rad. N.
longitude $\lambda_B = 3.12' = 3.2° = 0.056$ rad. W.

Radius of earth $= 6378$ km.

Course required for a direct rhumbline is

$$\tan\alpha = \left[\frac{\ln\left[\frac{\tan(\pi/4 + \varphi_B/2)}{\tan(\pi/4 + \varphi_A/2)}\right]}{\lambda_B - \lambda_A}\right]^{-1} = 4.533.$$

$$\therefore \alpha = 1.353 \text{ rad} = 77.56°.$$

Distance along the rhumbline is

$$S_R = R\sec\alpha\,(\varphi_A - \varphi_B)$$
$$= 6378 \times \sec(1.353) \times (0.8988 - 0.711)$$
$$= 5543 \text{ km}.$$

The great circle distance is

$$S_c = R\cos^{-1}\left[\cos\varphi_A\cos\varphi_B\cos(\lambda_A - \lambda_B) + \sin\varphi_A\sin\varphi_B\right]$$
$$= 5365 \text{ km}. \quad (\text{shorter by } 178 \text{ km}).$$

NOTE: In this problem and the next, the formula can give the wrong sign for α. The derivation of the formula for α has to be examined carefully for the cases where $\lambda < 0$ & $\lambda > 0$.

2). Let the intermediate waypoint be C.

longitude $\lambda_c = 38^\circ = 0.663$ rad.

The latitude at C is given by

$$\tan\varphi_c = -\frac{\left[\cos\varphi_B \sin\varphi_A \sin(\lambda_c-\lambda_B) + \cos\varphi_A \sin\varphi_B \sin(\lambda_A-\lambda_c)\right]}{\cos\varphi_A \cos\varphi_B \sin(\lambda_B-\lambda_A)}$$

$$= 1.3018$$

$\therefore \varphi_c = 0.915 \text{ rad} = 52.47^\circ$. N.

Course required for the first rhumbline is

$$\tan\alpha_1 = \frac{\lambda_c - \lambda_A}{\ln\left[\dfrac{\tan(\pi/4+\varphi_c/2)}{\tan(\pi/4+\varphi_A/2)}\right]} = 2.1.$$

$\therefore \alpha_1 = 1.126$ rad $= 64.53^\circ$.

distance along the 1st rhumbline $S_1 = \dfrac{\cancel{63}R\,|\varphi_c-\varphi_A|}{\cos\alpha_1} = 3023.91$ km.

Course required for the second rhumbline is

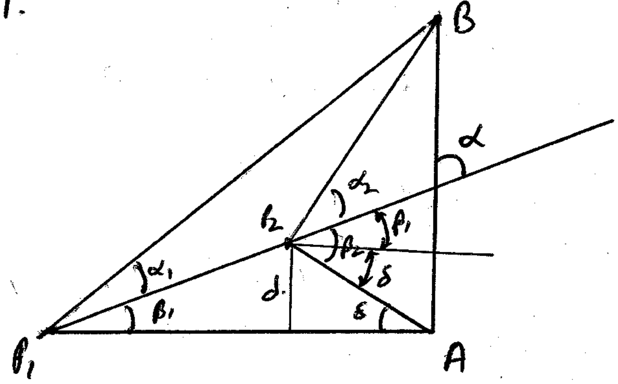
$$\tan\alpha_2 = \frac{\lambda_B - \lambda_c}{\ln\left[\dfrac{\tan(\pi/4+\varphi_B/2)}{\tan(\pi/4+\varphi_A/2)}\right]} = -23.087$$

$\therefore \alpha_2 = 1.614$ rad $= 92.48^\circ$

distance $S_2 = \dfrac{\cancel{6378}\,R\,|\varphi_B-\varphi_c|}{\sec\alpha_2} = 2392.29$ km.

$\therefore$ Total distance $= S_1 + S_2 = 5416.2$ km. (a saving of 126 km)

3) Soln. 1.



$\alpha_1 = 20.86°$, $\alpha_2 = 60.86°$

$\beta_1 = 9.14°$. $\beta_2 = 49.14°$.

$l(AB) = 1$.

$\angle BAP_1 = 90°$.

$$l(AP_1) = \frac{l(AB)}{\tan(\alpha_1+\beta_1)} = \frac{d}{\tan\beta_1} + \frac{d}{\tan\delta}. \quad \text{where } \delta = \beta_2 - \beta_1.$$

$$\therefore\ d = \frac{\cot(\alpha_1+\beta_1)\ \cancel{\cot\beta_1 \cot(\beta_2-\beta_1)}}{\cot\beta_1 + \cot(\beta_2-\beta_1)}.$$

$$l(P_1P_2) = \frac{d}{\sin\beta_1} = \frac{\cot(\alpha_1+\beta_1)}{\sin\beta_1\left(\cot\beta_1 + \cot(\beta_2-\beta_1)\right)}$$

$$= \frac{\cot 30°}{\left(\cos 9.14° + \frac{\sin 9.14°}{\tan 40°}\right)} = 1.472 \text{ km}.$$

Course $= \alpha = 90 - \beta_1 = 80.86°$.

(NOTE: α_2 is not used. Is there a way to use it?)