AE 695 – State Space Methods

Quiz 1, Monday, 17/9/07, 3:30pm-5pm, Open Notes, 20 marks

ONLY ONE'S OWN HANDWRITTEN NOTES ARE PERMITTED. PHOTOCOPIES, PRINTED MATTER ARE NOT ALLOWED.

1. Let $\mathcal{V}_1 = \{P \in \mathbb{R}^{3\times 3} : P = P^{\mathrm{T}}\}$ denote the set of 3×3 real symmetric matrices and $\mathcal{V}_2 = \{A \in \mathbb{R}^{3\times 3} : A^2 = A\}$ denote the set of 3×3 real *idempotent* matrices.

   (a) One of $\mathcal{V}_1$ and $\mathcal{V}_2$ is a vector space over $\mathbb{R}$ under ordinary matrix addition and scalar multiplication. Which one? Explain briefly. (2)

   (b) Let $\mathcal{V}$ stand for the vector space among the sets $\mathcal{V}_1$ and $\mathcal{V}_2$ (that is, let $\mathcal{V}$ denote your answer to part a) above). What is the dimension of $\mathcal{V}$? Find a basis for $\mathcal{V}$. (3)

   (c) Define two real-valued functions $f_1 : \mathcal{V} \to \mathbb{R}$ and $f_2 : \mathcal{V} \to \mathbb{R}$ by $f_1(A) = \text{trace A}$ and $f_2(A) = \det A$ (where "det" denotes the matrix determinant). Which of f_1 and f_2 is a linear operator? Explain briefly. (2)

   (d) Let $\mathcal{L}$ denote the linear operator among f_1 and f_2 (that is, let $\mathcal{L}$ denote your answer to part c) above). What are the range, kernel, rank and nullity of $\mathcal{L}$? Find a basis for kernel $\mathcal{L}$. (4)

2. Show that the functions e^t, te^t and t^2e^t are linearly independent on the interval $[0, 1]$ over the field $\mathbb{R}$. (3)

3. Let $\mathcal{V} = \mathbb{R}_3[s]$ be the set of polynomials in s having real coefficients, and degree 3 or less. Let $\mathcal{L} : \mathcal{V} \to \mathcal{V}$ be the operator that maps any given polynomial p in $\mathcal{V}$ to the polynomial $p'' + 3p' + 2p$, where $'$ indicates differentiation with respect to s. Find the matrix representation of $\mathcal{L}$ in the basis $\{1, s, s^2, s^3\}$ for $\mathcal{V}$. Using the matrix representation, or otherwise, find the rank of $\mathcal{L}$. (4)

4. Is the set of real invertible $n \times n$ matrices a field under ordinary matrix multiplication and addition? Explain briefly. (2)