AE 695 – State Space Methods

Quiz 2, Friday, 26/10/07, 6:30pm-8pm, Open Notes, 15 marks

ONLY ONE'S OWN HANDWRITTEN NOTES ARE PERMITTED. PHOTOCOPIES, PRINTED MATTER ARE NOT ALLOWED. This paper is printed on BOTH sides.

1. Consider the matrix-valued function $F : \mathbb{R} \to \mathbb{R}^{2\times 2}$ given by

$$F(t) = \begin{bmatrix} e^t & e^{-t} \\ e^{-t} & e^t \end{bmatrix}.$$

Does there exist $A \in \mathbb{R}^{2\times 2}$ such that $F(t) = e^{At}$? Explain briefly. (2)

2. Is the following true? Explain briefly. (2)

$$e^{At} = e^{3t} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

3. Find the state response of the system

$$\begin{aligned} \dot{x}_1 &= 2x_2, \\ \dot{x}_2 &= 2x_1, \end{aligned}$$

to the initial condition $x(0) = [1 \ \ 0]^{\mathrm{T}}$. (3)

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4. Verify that 2 is an eigenvalue of the matrix A given below. Find the algebraic and geometric multiplicities, and Jordan block corresponding to this eigenvalue. (4)

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ -1 & 0 & 3 & 0 \\ -1 & 0 & 1 & 2 \end{bmatrix}.$$

5. Suppose the matrices A, P_1 and P_2 are as given below. Ascertain whether the matrices $A^{\mathrm{T}}A$, $A + A^{\mathrm{T}}$, $A^{\mathrm{T}}P_1 + P_1A$ and $A^{\mathrm{T}}P_2 + P_2A$ are positive definite, negative definite, positive semidefinite, negative semidefinite, or indefinite. (4)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}, P_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 11 & 1 \\ 1 & 5 \end{bmatrix}.$$

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