AE 695 – State Space Methods

Quiz 4, Thursday, 09/11/06, 3:30pm-5pm, Closed Notes, 15 marks

1. Show that the state space system having

$$A = \begin{bmatrix} 0 & 1 \\ -ab & (a+b) \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = [-c\ 1],\ D = [0]$$

is a realization of the transfer function $(s-c)/[(s-a)(s-b)]$. Is the realization controllable and observable in general? Is the realization controllable and observable if $c = a$? Find the transfer function and a minimal realization for the system in the case $c = a$. (6)

2. Statements (a)-(d) below concern a linear input-output system that has a (A, B, C, D) state space realization with the following properties.

   - The matrix A is 4×4.
   - There exists a vector $x_0 \in \mathbb{R}^4$ such that $Cx_0 \neq 0$ and $x(t) = e^{2t}x_0$ is an initial condition state response of the realization.
   - The poles of the transfer matrix are $-3 \pm 0.4\jmath$, -4.

   Are the following statements true, false, or undecidable from the information provided? Support your answer in each case with a brief explanation of not more than five lines. Explanations that are wrong or incomplete will receive no credit. (6)

   (a) The realization is asymptotically stable.

   (b) The realization is observable.

   (c) The realization is controllable.

   (d) If $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ is a realization of the same system, where $\hat{A}$ is 3×3, then the pair $(\hat{A}, \hat{B})$ is controllable.

3. State if the state space system having the following matrices is controllable and/or observable. (3)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},\ D = 0.$$