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Review of supersonic boundary layer instability and transition models

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A literature review on both the theoretical and experimental work on the stability and structure of supersonic boundary layers has been made. New trends of theoretical research such as the parabolized stability equations and transition modelling have been surveyed. Several types of breakdowns of incompressible and compressible boundary layers have been discussed.

1 Introduction

Extensive research has been carried out to study the stability and transition of incompressible boundary layers, but the compressible counterpart has not received adequate attention. The effect of compressibility on the structure of boundary layer needs study. The present effort is to study the stability behaviour of the supersonic and hypersonic boundary layers in the presence of adverse pressure gradients and free stream turbulence. Compressible boundary layers have dominantly 2-D instabilities of Tollmein-Schlichting (TS) waves in the early transition, which couple with subharmonics in resonance and non-resonance mechanisms. Dominant transition structures are the Λ (hairpin) vortices (3-D) in staggered (peak-valley) type and commonly occurring with large amplitudes or in aligned pattern when growth rate is supressed by external forcing, implicitly (complaint coatings) or explicitly (suction, favourable pressure gradients, cooling, etc.). The aim of the present study is to see the occurrence of 3-D disturbances (Λ vortices) and predict transitional criteria based on spatial growth rates in developing flows by e^N criteria for 2-D and 3-D boundary layers. As the direct numerical simulations (DNS) studies are confined to the low Reynolds number and simpler geometries, the success of viscous flow around a body largely depends on the modelling of laminar-turbulent transition. Compressible boundary layer transition study is very important in the design of future hypersonic vehicles. Various works carried out on incompressible and compressible boundary layers relevant to our present effort have been discussed in this paper.

2 Survey of primary and secondary instability studies in boundary layers

Subsonic disturbances in compressible unbounded (free) shear layers are 3-D primary instabilities unlike the 2-D nature in boundary layers and confined shear layers. These are the modifications of vorticity modes for incompressible flows. For supersonic flow, up to the local Mach number of 2.0, the disturbances are similar to those of subsonic disturbances. For Mach number greater than 2.0, these disturbances are 2-D and consist of both primary and secondary instabilities. They are complicated modes of vorticity, entropy and pressure (shock) waves.

Stability research in incompressible boundary layers (BLs) dates back to Prandtl[1,2], with first experimental confirmation by Schlichting[3] , and later by more correct experimental confirmation by Schubauer and his coworkers[4,5] using hot wire anneometry. The 2-D nature of Tollmein Schlichting (TS) waves preceding transition was discovered by them. Dhawan and Narashima[6], Sato[7] and Klebanoff *et al.*[8,9] confirmed experimentally a critical Reynolds number of 600 for sand-paper-type distributed roughness, and also identified the 3-D nature of transition streaks (Λ or hairpin vortices), together with the turbulent spots. Effects of free stream turbulence and wall roughness on the transition characteristics were also studied by them. Klebanoff and Tidstorm[9] experimentally confirmed that instability waves precede transition and that instability amplification is a precursor to transition in a flow behind a 2-D roughness element. With the development of compressibility theory by Langerstorm *et al.*[10], theoretical stability studies were initiated by Lees and Lin[11] and many others[12-16]. These works mostly consist of development of linear

stability theory (both inviscid and viscous) using parallel flow, normal mode assumptions and numerically integrating the stability equations. Experimental confirmation of critical Reynolds number for subsonic and transonic flows was given by Laufer[17-19] and Narasimha[20] . Van Dreist and Boison[21], Van Dreist and Blumer[22], and many others[23-25] extended the study to supersonic and hypersonic flows. With increase in Mach number, higher modes known as Mack-modes were determined. Also, in the absence of definitive transition theory, various criterion to mark the onset of transition were used. Van Dreist and Boison[21] used the ratio of Reynolds stresses to mean viscous stresses equated to a certain critical value as a criteria to locate transition inception. Laufer and
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(1) Vrepalavich[18], Laufer[19] and Pate[23] on the other hand used a correlation to demarcate transition based on irradiated sound level from wall BLs. Another approach is to use linear stability theory, though it suffers from the limitation of being linear. In transition research, although, the critical Reynolds number is, at times, a useful quantity in stability theory, it is too remote from transition in most instances to serve as an indication of transition dependence on the mean flow. The amplification of the disturbances is a decisive factor and not its initial point of instability. Calculations of amplitude growth of constant frequency disturbances led to the well known e^9 criterion of Smith and Gamgeroni[26] and Van Ingen[27], later modified to e^{10} by Jaffe *et al.*[28] and used by Mack[16] to correlate the numerical stability results.

Most experimentalists have approached the BL transition problem by artificially exciting the flow with relatively 2-D small amplitude, single frequency excitation devices, such as vibrating ribbons and acoustic speakers. They often went to great lengths to reduce the background disturbance levels to an absolute minimum in order to make the external forcing in their experiments as small as possible. The initial disturbance should be harmonic in time and 2-D, which is well described by linear stability theory for the low Mach numbers at which most experiments have been carried out. This 2-D linear behaviour can persist over long streamwise disturbances when excitation levels are sufficiently small, but eventually become 3-D, as evidenced by the appearance of Λ shaped structures in experiments where smoke flow visualization is used. These structures, which are arranged in rows, can either be aligned or staggered in alternating rows. The unstaggered arrangement which was first observed by Klebanoff *et al.*[8], ~~is now commonly referred to as peak-valley splitting~~. The staggered arrangement which usually appears at low excitation levels, is believed to be the result of resonance triad interaction between a pair of oblique subharmonic modes (which originate from background disturbance environment) with the basic fundamental 2-D mode. Transition research needs to address the receptivity problem, which is summarized in the reveiws by Reshotko[29] and Goldsetin and Hultgren[30] .The detailed mechanism of how the BL ingests external acoustic waves or turbulent fluctuations in a given geometry with inadvertent roughness and vibrations of the solid boundary is denoted as receptivity. It also involves understanding of the mechanisms by which a forcing disturbance of a given frequency and prescribed phase velocity excites a free disturbance of same frequency but different phase velocity. From such studies it became clear that strong inhomogeneities of the mean flow lead to local regions, where disturbances aquire the TS wavelength and couple locally with the TS wave. Also, factors which affect transition may be controlled for effective drag reduction. To laminarize the flow, it is seen that suction, cooling in air, heating in water and favourable pressure gradient tend to make d^2U / dy^2 negative and aid the laminarization process. It is observed that under the presence of very small adverse pressure gradient, linear growth rates will be small, of the order of, pressure gradient squared, and the instability wave will have a well defined critical layer, but will be of a non-equilibrium (or growth dominated) type rather than the equilibrium (or viscous dominated) type associated with TS waves.

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(2)

The instability dynamics is quite different at supersonic speeds. This involve linear inviscid as well as viscous modes. At high Mach numbers, the most growing linear mode is the oblique mode. Thus, to simulate the transition phenomena at high speeeds, one needs to artificially generate the oblique modes, generally done in pairs which are progressive in the streamwise direction but stationary (standing modes) in the spanwise direction. This is done by generating pairs of oblique vortices of opposite rotation. The high speed Mack modes (supersonic type) occur theoretically when the free stream flow has a local Mach number, $M > 2.2$, but it is not experimentally detected until $M > 4.0$. In contrast to the first or TS mode, the higher modes are destabilized by cooling[16,31]. On the other hand these modes are stabilized by favourable pressure gradients[32], or by suction[33-35]. In supersonic flows, studies on the effect of nose bluntness (leading edge) on stability has been done by many workers[36-39]. Pruett *et al.*[40] extended such studies to hypersonic flows. It is observed that for a blunt flat plate, upstream of the location where BL swallows the layer coming through the strong part of the bow-shock wave (entropy layer), both BL and the shock are seperately unstable in a generalized inflectional sense. After the BL swallows the entropy layer, the BL profiles asymptotes to those of the sharp leading edge and their stability characteristics follow suit. Within the swallowing region, the stability charecteristics are also affected by the fact that the shock layer of the flow at the edge of BL is non-uniform in the vertical coordinate y. This stabilizes the first mode but destabilizes the second mode. For hypersonic flows, the thickness of the shock layer is small; hence, the shock layer is affected by free stream disturbances. Hence, one cannot have normal mode eigen-functions which decays exponentially at infinity, but must have boundary conditions giving finite amplitude disturbances at infinity. Such hypersonic stability models have been proposed by Cowley and Hall[39] using triple deck theory[41] extended to compressible flows. The use of impulse response techniques to analyse stability of BLs may be accounted for in the pioneering work of Criminale and Kovasznay[42] and Gaster[43,44] with the experimental verifications given by Gaster and Grant[45].

3 Special study of theoretical work on non-linear stability analysis

Most of the earlier discussion in BL stability was focussed on the stability developments. Weakly and fully non-linear analysis was done on two fronts— (i) to account for breakdown of parallel flow assumption and (ii) to develop models describing resonance triads to explain the aligned (Λ) and staggered (hairpin) vortices, representing 3-D disturbances. Non-parallel stability problems for incompressible BLs have yielded to different asymptotic methods. Bouthier[46] and Gaster[43] gave a reliable and effective successive approximation procedure to describe the non-parallel linear growth of TS waves in developing BLs. At zeroth order, the Orr Sommerfeld equation (OSE) is obtained by this procedure. Gaster[43] showed that this approach was capable of improving significantly the agreement between the theoretical and experimental results. An alternative approach based on more formal asymptotic methods was developed by Smith[47]. This asymptotic expansion was developed in an appropriate manner to capture either the lower or the upper branch of the stability curve. Typically it was found that the lower branch structure, based on triple deck theory, is most successful in reproducing the available experimental results. Since the earlier mentioned successive approximation method is capable of producing both branches of the the neutral curve, it has been argued elsewhere[43] that this is the most efficient method of determining growth rates in developing BLs. This argument has been reinforced by the development of high speed computers which make the OSE solution a routine task. However, since most important problems in BLs stability theory concern non-linear effects, and the multiple deck solutions provide the only rational self-consistent framework for non-linear studies in developing BLs, it is clear that, if anything other than linear growth rates is required, the successive approximation procedure is of limited use. Multiple deck theories have also been extended to compressible BLs. However, it is seen that the Cowley and Hall's[39] calculations for stability of wedge flows, in presence of shocks, cannot be tackled using successive approximation techniques. For many years, non-linear stability in hydrodynamics was almost exclusively based on ideas on the expansion procedure of Stuart[48] and Watson[49]. This procedure uses underlying procedures of strained coordinates to formulate the multiple scales to describe how the energy in a wave cascades into the harmonics, which enables us to calculate the amplitude of the wave at Reynolds numbers close to the neutral one. The application of this method to BLs gets complicated due to BL growth. It was not until Smith[47] who applied triple deck theory to the problem, that a self consistent derivation of the amplitude evolution equation of the TS wave was available. This formulation is valid near the lower branch of the neutral curve. In subsonic compressible BLs, inviscid modes are possible causes of instability. The wavelength of these modes typically scales on the BL thickness, so that a multiple scale approach leads to *Rayleigh*'s equation at zeroth order in an asymptotic description of these modes[50]. However, in high speed flows inviscid as well as viscous modes are important. However, asymptotic investigations for inviscid compressible linear theories for the development of 3-D structures have an intrinsic shortcomming in that they are necessarily dominated by long (viscous) time scales, (characteristic of linear theory), whereas the effects they are intended to elucidate, develop on fast (inertial) time scales. Fully non-linear simulations as given by Orszag and Patera[57], Herbert[58] and El-Hardy[59] reveal that fast, short wavelength 3-D instabilities with the same periodicity of the basic wave, are a precursor in the transition regime developing to turbulence. The BL experiments show that TS waves of sufficiently small amplitude harmlessly grow and decay. At larger amplitudes, TS waves form 3-D structures in the form of Λ vortices (aligned configurations) of subharmonic modes or combination modes. The occurrence of 3-D structures is a necessary prerequisite, but not an assurance for breakdown. Breakdown occurs only under some specific modulations to the TS wave. The H- type breakdown in incompressible BLs was proposed by Herbert, based on an extension of linear secondary instability theory of Herbert and Markovin[60] to the finite and large disturbances. The work of Herbert and Markovin[60] essentially consists of a non-resonant type excitation of a 2-D TS wave (α , 0) with a longitudinal vortex mode (0, β). However, it lacked the non-parallel considerations, and essentially borrowed ideas from Maseev[61] . Herbert's work involve study of large secondary disturbances, leading to H- type breakdown, due to interaction of 3-D secondary instabilities, parametrically excited by a 2-D TS wave in BLs. This is a fully non-linear type analysis and Floquet theory forms the natural basis of such studies that was applied earlier in the work of Orszag and Patera[57]. The breakdown associated with this resonance triad is called as H-type (or Herbert's type) breakdown and is associated with the staggered configuration of vortices observed experimentally by Saric and Thomas[54] and Kachanov and Levchenko[53]. In Floquet theory, one considers parallel flow with a superposed wave of fixed amplitude and period together with a coordinate system moving with the wave (at wave speed). The linear stability[52] was governed by a Floquet system of differential equations and periodic coefficients. For 3-D, incompressible BLs on swept wings, El Hardy's model[60] can predict large amplification for the subharmonic wave depending on the initial spectrum of amplitudes and phases of the triad components. The nature of subharmonic instability development and breakdown for compressible BL flows is not well understood, but has been studied recently by El-Hardy[62], Masad and Nayfeh[35], Ng and Erlebacher[63]. The most unstable disturbance in compressible flows is either a 2-D disturbance, for M<1.0, or an oblique mode for M >1.0. Ng and Erlebacher[63] applied Floquet theory modified for primary oblique waves, but nothing was discovered about the types of breakdown except that secondary instabilities are formed and the compressibility has an overall stabilizing nature on the subharmonic instabiltiy modes. Masad and Nayfeh[35] found numerically that increasing the Mach number stabilized the most unstable subharmonic waves. The decrease in subharmonic amplitude with the increase of Mach number might affect the production of streamwise vorticity, resulting in slowing or delaying the flow breakdown to turbulence. Also, it is found that the maximum of root mean square values of the secondary harmonic stream-function eigen-functions (representing kinetic energy of secondary disturbances) moves away from the wall following a critical layer as the Mach number increases. This suggests that the exchange of energy among secondary disturbances, the mean flow and the fundamental wave takes place around this location at different Mach numbers. A discussion on the present status of experimental results of boundary layer stability is summarized by Mueller[64] and the results based on a theoretical foundations is summarized by Reed *et al.*[65] and Herbert[66]. Another concept which has aided stability research in BLs (and proposed only a decade ago) is the application of parabolized stability equations (PSE). A review of studies in this area is given by Herbert[66].

4 Special survey of experimental work

The crucial ingredients of turbulent flow usually originate in the transition region, viz. vortex interactions, leading to 3-D vortex structures and transition studies help in understanding the mechanism from where they originate in turbulent flows. Mueller's work[64] gives a historical note on the development of smoke visualization and hot-wire anneometry and also present the respective experiments using these instruments in the study of transition research. Brown[67,68] initiated research using a spinning tangent-ogdive nose axisymmetric model and found striations corresponding to cross flow vortices due to inflectional instability. Later, when he used a non-spinning model, he was first to observe and photograph TS waves and their breakdown using smoke visualization. These photographs revealed the TS waves (2-D) deformed to 3-D forms and formed vortex loops which Brown called trusses and then broke down into turbulence. The transition process occurred naturally as a result of free stream turbulence in a low turbulence wind tunnel (i.e. turbulent intensity less than 0.1%) and not caused by vibrating ribbon or tape. Hama *et al.*[69] did flow visualization study in water of 3-D features of transition (found earlier) by vibrating ribbons. Trip wire caused 2-D vortices to be shed and deformed into 3-D vortex and vortex loops which get stretched and deformed as they moved downstream and eventually into turbulence. Further research by Brown[14] (over the next decade with hot-wire anneometry) over a body of revolution helped him to characterize the natural transition in 4 stages, namely, (i) formation of a set of 2-D waves, (ii) 3-D deformation of these waves, (iii) region of vortex trusses (or hairpin vortices) and (iv) breakdown of vortex trusses into regions of turbulence. Laminar region shows breakdown followed by reappearance of 2-D waves. These vortex trusses have aligned or staggered attachment. Brown[14] found that anything that delays formation of Λ -vortices causes the staggered pattern to form. (e.g. lower pressure gradients, lack of artificial forcing, etc.). The K-type breakdown study of Klebanoff *et al.*[8] with vibrating ribbon on flat plate showed that in low disturbance environments, the forerunner of transition to turbulence is the 2-D TS instability wave. Their hot-wire experiments showed that 2-D wave stage was followed by an ordered (aligned) pattern of Λ vortices. Peaks followed peaks, and valleys followed valleys in K-type breakdown. Markovin[25] presented a critical evaluation of the state of understanding of transition in 1969. This comprehensive evaluation, however, did not clarify the understanding of the origin of aligned and staggered Λ vortex pattern. While the research of Brown[14] and others[67-70] showed the way in transition studies, the research of Head[71] led the way in the visual study of structure of turbulent BLs. Smoke visualization studies came into the picture in late 1970's, and in early 1980's it became an important tool (for more details refer Mueller[64]). In 1977, Kachanov and Levchenko[53] first reported measurements of subharmonic signals in their hot-wire study of the non-linear development of a wave in BL. Thomas[72] observed frequency halving of hot-wire anneometer signals which was consistent with the staggered pattern Λ of vortices with wavelength 2 λ_x. These quantitative indications of subharmonic mode were clarified later by smoke visualization. Tani[73] discussed the theoretical work of Craik[56] and others on non-linear interactions among 2-D and 3-D waves with two frequencies. However, he was inconclusive about the problem. Kegelman[74] and Mueller[64] used different models of axisymmertic bodies and at least 3 different modes of transition were observed over the range of conditions given as follows: (i) the 2-D TS waves may stop growing in locally favourable pressure gradients, (ii) for somewhat larger amplitudes of TS waves, a secondary instability takes place leading to a staggered Λ vortex pattern and (iii) with still larger amplitudes of TS waves the Λ vortices are aligned. Kegelman[76] was able to change the staggered patterns to aligned patterns by acoustic enhancement of the amplitude of TS waves, all other conditions remaining unchanged. The staggered Λ vortex formation was a puzzle ever since they were observed by Brown and others[67-70]. These were reorganized by the fact that the visible wavelength suddenly doubles from the upstream TS wavelength of the growing TS waves. Craik[56] in 1971 presented a theory for a flat plate which described the mechanism of non-linear traid resonance (referred to as C-type breakdown) which appeared to explain some of the staggered Λ vortex formation. Herbert[51,52,58,60] recently described another subharmonic mechanism consistent with the visual and hot-wire information, called as H-type breakdown. The combined hot-wire and smoke-wire studies of Saric and Thomas[54] and Kachanov and Levchenko[53] showed the difference between C-type, H-type and K-type breakdowns. Two excellent detailed reveiws of secondary instabilities in boundary layers are given by Thomas[72] and Herbert[75]. When 2-D TS waves of frequency, f, grow, towards the end of the passage of amplification of linear stability theory, it passes energy, non-linearly to skew vorticity waves of frequency $f/2$. Such skew waves are observable at the legs of Λ vortices. According to Saric and Thomas[54] the threshold for K-type breakdown is as low as 0.003 U_∞ involving skew waves. Herbert's H-type breakdown have Λ vortices of short spanwise scale and the threshold value is lower than that of K-type breakdown. Hairpin

5 U_∞ infinity

vortices are seldom detectable, but it is topologically same as K-type breakdown. At intermediate Reynolds number, the Craik or Herbert breakdowns are found. Kegleman and Mueller[76] found that at high frequencies, Λ vortices appeared in staggered pattern when sound waves are introduced at low amplitudes and an aligned pattern when sound was introduced at high amplitudes. These breakdown into spikes–short lived, large amplitude pulses, which are deterministic in nature (turbulent spots). At low frequencies the Λ vortices always appeared in aligned fashion. The breakdown of aligned vortices into intermittant, large amplitude fluctuations (turbulent spots) represents K-type breakdown. This staggered pattern is also related to the N-type breakdown phenomena where subharmonic resonance plays a decisive role . The subharmonics have a detuned frequency, ½ of that of the primary wave. The staggered pattern vortices breakdown into random fluctuations into turbulence via secondary instability mechanisms. These acoustic responses were given to make the 3-D response of TS waves sensitive to both frequency and amplitude.

It may be noted that Liepmann[77] was the first to convert linear stability results into a transition criteria for practical applications. His criteria is based on the ratio A/A_0 , where $A(x)$ is the wave amplitude as it evolves downstream and A_0 is the unknown but small amplitude at the onset position x of instabilty. Based on Gaster's work[78], the phase velocity c was replaced by the group velocity c_g to correctly convert the temporal to spatial growth rate. The amplitude ratio is given by ($-\alpha_i$ is the spatial growth rate)

$$\sigma(x;\omega) = \ln(A/A_0) = -\int_{x_0}^{x} \alpha_i(\xi,\omega)\, d\xi \qquad \dots (1)$$

Liepmann[77] related the critical value of amplitude ratio to the skin friction coefficient C in laminar flow. Smith and Gamberoni[26] and Van Ingen[27] later found that transition in wind tunnel tests correlated well with values of N factor, $[N = \max\ \sigma(x_{tr}, \omega)]$, N taking the values between 6 and 9 and $A_{tr}/A_0 = e^N$. The e^N method for 2-D TS waves has been extended to compressible and 3-D boundary layers, and to other types of instabilities such as Gortler vortices on concave walls, the cross- flow instability in 3-D flows over infinite swept wings[79] and Mack's higher modes in supersonic flows. Today the e^N method is the mainstay of transition prediction in aerodynamic design.

5 Special survey of parabolized stability equations

Parabolized stability equations [66] (PNE) have opened new avenues to the analysis of streamwise growth of linear and non-linear disturbances in slowly varying shear flows such as boundary layers, jets and wakes. Growth mechanisms include both algebraic transient growth and exponential growth through primary and higher instabilities. In contrast to the eigen-solution of traditional linear stability equations, PSE solutions incorporate inhomogeneous initial and boundary conditions as the numerical solutions of Navier-Stokes equations do, but they can be obtained at modest computational expense. The PSE codes have developed into a convenient tool to analyse basic mechanisms in boundary layer flows. The most important area of application, however, is the use of PSE approach for transition analysis in aerodynamic design. Together with the adjoint linear problem, PSE methods promise improved design capabilities for laminar flow control systems. Flow non-parallism was tackled by Gaster[43] and Boulthier[46] where WKBJ theory (or multiple scales) formed the intermediate step leading to a partial differential equation (p.d.es) similar to parabolized Navier-Stokes equations, except for additional terms containing frequency and wavenumbers. Gaster went further to reduce the p.d.es. to more tractable ordinary differential equations. These final steps are unnecessary as they severely restrict the scope and validity of the resulting equations. The intermediate p.d.es. were used by Herbert[66] to validate Gaster's results and encouraged the potential of the PSE (or earlier mentioned intermediate p.d.es) for research and engineering applications. The parabolized stability equations are given as follows:

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$L\,\mathbf{q} = 0$, L being the linear stability operator, we have

$$(L+\varepsilon L')\mathbf{q}+\varepsilon M_1\frac{\partial \mathbf{q}}{\partial \varepsilon}+\varepsilon M_2\frac{\partial \mathbf{q}}{\partial \rho} = \mathbf{r}-\varepsilon\left[\frac{d\alpha}{d\xi}N_1\mathbf{q}+\frac{d\beta}{d\rho}N_2\mathbf{q}\right] \qquad \dots (2)$$

where the operator L, L' , M_1 , M_2 , N_1 , N_2 , act in y only. The term in square brackets can be neglected because N_1 and N_2 originate from viscous terms of ~ 0

(Re^{-1}). The function $\mathbf{r} = \mathbf{r}(y)$ on the right side represent non-linear terms that need further discussion which may be found in Herbert's reveiw[66]. At limit $\varepsilon \rightarrow 0$, we get the governing equation of linear stability theory. The use of marching technique for solving PSE is permitted only if the stability problem is governed by downstream propagating information. Hence PSE approach is valid for convectively unstable flows. The PSE equations are not really parabolic and similar to parabolized NSE. The PSEs exhibit weak ellipticity. The PSE account for the history of the disturbance and for streamwise variation (non-parallel effects) of the flow. Non-parallelism is partially included even if $L' = 0$. The operator L requires small-velocity terms that are not provided by most BL codes and are difficult to retrieve from numerical data ($L' = 0$ governs parallel flows, 2-D flows, quasi 3-D flows on swept cylindrical bodies such as infinitely swept wings). This special case has been studied by Bertolitti[80]. For $\alpha = 0$, the PSEs are closely related to the parabolic equations introduced by Hall[81] for the analysis of Gortler vortices. The PSE provides a connected physical solution along the marching path and spatial amplitude growth curves σ (x; ω,β) directly for use in the e^N method. Long waves and surface curvature can be treated consistently. The initial -boundary problem is suitable for analysing exponential as well as transient growth of disturbances. The PSE can be solved with disturbed boundary conditions. These inhomogeneous conditions include freestream turbulence or roughness that does not affect the basic flow. The PSE allows for an inhomogeneous RHS term $\mathbf{r}$ that may originate from non-linear terms ($\mathbf{V}'$. ∇)$\mathbf{V}'$ in combination with other forcing functions. Systems of PSE for different frequencies ω_n and wavenumbers β_n can be solved simultaneously to trace the non-linear evolution of single mode or interactions of different modes. For these reasons, the PSE approach is suitable for simulating the early stages of transition. The PSE can compete with direct numerical simulation (DNS). The PSEs have been extended to compressible flows[82]. With a restricted number of modes, the PSE method allows fast transition simulation up to the breakdown stage where significant changes in skin friction (or heat transfer) occur. The marching solution terminates typically, because the iterative update of wavenumber or the iterative solution of the non-linear system for different modes fails to converge. On comparing Gaster's[43] method of successive approximation with Herbert's PSE method[66], we may say that the former method[43] is non-parallel by successive approximation but the latter method[66] is non-parallel and fully non-linear. Further discussion of the present status of PSE applications and future developments are summarized by Herbert[66].

6 Special survey on laminar flow control

A strong international interest in the problems of stability and transition in wall-bounded shear layers exists in connection with the design of gas-turbine-engine blades and vanes, submarines and torpedoes, subsonic and supersonic civil transports and fighters, hypersonic and re-entry vehicles. Understanding transition is necessary for accurate prediction of aerodynamic forces (lift and drag) and heating requirements. Moreover, delaying transition by various techniques of laminar flow control (LFC) generally results in lower drag and, therefore, higher fuel efficiency[29,83-93]. It has been estimated that if laminar flow could be maintained on the wing of a large transport aircraft, a fuel saving of up to 25% would be obtained [94,95]. Of interest to turbulence community is the fact that BL flows are open systems, strongly influenced and dependent on freestream and wall conditions. Breakdown to turbulence has been well documented to vary considerably when operating conditions change, as evidenced by the works of Saric and Thomas[96] and Corke[97]. The transition process then provides vital upstream conditions from which the downstream flowfield evolves, and it is reasonable to imagine that different transition patterns give rise to different turbulence characteristics downstream. To control skin friction or heat transfer one either modifies the turbulence structure or prevents the BL from becoming turbulent by limiting the growth of disturbances. The latter technique is known as laminar flow control (LFC) and is efficient only in low-disturbance environments. For engineering applications, passive control (i.e. static manipulation of the boundaries of the flow field) seems more feasible than active feedback control through wave cancellation.. As linear stability can easily be calculated, passive control schemes are usually designed based on linear theory. However, since the initial conditions (receptivity) are not generally known, only relative comparisons between control

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schemes are possible. For 2-D BLs, the growth of linear disturbances is weak and occurs over a viscous length scale and can be modulated by pressure gradients, mass flow, temperature gradients, etc. As the amplitude grows, 3-D and non-linear interactions occur to form secondary instabilities[58]. At this point, growth is very rapid (now over a convective length scale) and breakdown to turbulence occurs quickly. On the other hand, for 3-D BLs (e.g. swept wings) and Gortler problems (concave surfaces), non-linear distortions of the basic flow may occur early due to the action of primary instability. These flows are characterized by extensive distance of non-linear evolution with eventual saturation of the fundamental disturbance, leading to strong amplification of very-high-frequency inflectional instabilities and breakdown[98,99]. In these situations, linear stability theory must be applied with care and caution, and other techniques such as PSEs[66] and DNS may be found appropriate because they can account for non-parallel flows, non-linear stability, non-linear interaction of modes and mean flow distortion. At times, the initial instability can be so strong that the growth of linear disturbances are bypassed and turbulent spots and secondary instabilities occur which quickly make the flow turbulent (occur when roughness and free-stream turbulence are large). Linear stability theory fails in such cases used in transition prediction schemes. The review of BL transition research by Mack[100] is worth mentioning.

7 Special survey on transition models

The basic fluid dynamical problems associated with the transition of laminar to turbulent boundary layers still remain poorly understood, although the wide recognition of scientific and technological importance of the subject has led to extensive research on both the experimental and analytical front. Initially the boundary layer on any surface remains steady and laminar up to a certain distance from the leading edge and then it starts exhibiting unsteady behaviour involving 2-D TS wave and 3-D secondary instabilities further downstream as the Reynolds number for instability increases. As the 3-D disturbances grow, a stage is eventually reached when the flow breaks down with the appearance of intermittent turbulent fluctuations coinciding with what we shall call the "onset of transition". Several reviews highlighting various aspects of transition appeared in the literature[75,101-103]. It is clear from these studies that there is a lack of argument on the precise stages in the flow development, leading to transition. On the order in which they occur and on the factors that influence them. No general theory could be made to predict the process of transition. In this paper various classes of transition models for the computation of viscous flows are discussed.

The existing transition models for boundary layers can be broadly classified into three categories:

(i) Linear combination[6,104-110]
(ii) Algebraic[111-116]
(iii) Differential[117-123].

In the linear combination models, the mean flow is determined by combining mean laminar and turbulent velocities in proportion determined by intermittency γ. All models in the linear combination class require methods for carrying the following tasks: (i) calculation of the laminar boundary layer, (ii) estimation of mean flow parameters in fully turbulent boundary layer starting from an arbitary station in the flow, (iii) prediction of the location of the onset of transition, and (iv) the intermittency distribution in the transition zone. The methods differ only in the manner in which the tasks are performed.

In algebraic models, the time averaged Navier Stokes equation is taken with an appropriate algebraic model for Reynolds stress. The turbulent viscosity is gradually turned on in the transition zone in proportions determined by intermittency. For example, an effective total differential (including viscosity) in the flow may be taken to be $\nu = \nu + \gamma \nu_T$, where ν_T is the eddy diffusivity. The transitional intermittency γ has been obtained separately, thus requiring information on onset location and transition zone length. Though the use of eddy viscosity can be justified on consideration of large eddy equilibrium, the concept suffers from the well known limitation of all gradient transport theories. Nevertheless when properly used, eddy viscosity can provide useful estimate of certain boundary layer characteristics.

Differential models also directly tackled Reynolds average equations of motion with either one- or two-equation turbulence closure models. In the one-equation model, the turbulent velocity scale is determined by solving a partial differential equation describing the transport of turbulent kinetic energy (k) and the length scale (L) is determined from some algebraic relations and the eddy viscosity is obtained from $\mu_t = \rho / k\ L$. In the two- equation model, we solve for the turbulent velocity and length scale by solving cancell 7 the transport equation of turbulent kinetic energy ~~and the rate of dissipation of turbulent kinetic energy~~ and the rate of dissipation of turbulent kinetic energy (ε), the so called k-ε model[124-125]. Although the prediction of boundary layer parameters in these models does not require any specific definition of the beginning and end of transition, the range over which the turbulent energy increases from initially low values to the final turbulent values can be considered to correspond to the transition zone. However, for triggering transition these models require some initial disturbance in terms of an initial profile of turbulent energy or a source term in the energy equation.

8 Summary

A review on both the theoretical and experimental work on the stability and structure of the supersonic/hypersonic BLs under the influence of adverse pressure gradient and free stream turbulence has been presented. It has been observed that the stability studies of the supersonic BLs have not received adequate attention compared to its incompressible counterpart. On comparing the two main theoretical works in incompressible non-linear transition it is found that the Craik's work[56] involves resonant forcing and is weakly non-linear, while Herbert's[58] work is a non-resonant forcing (using a detuning parameter) where resonance occurs in the secondary level with non-linear interaction (analysed using Floquet theory but restrained to parallel flow assumption). The staggered pattern obtained from this $^1/_2$ factor-detuning-frequency represents the N-type breakdown of Kachnov[126]. The K-type breakdown[8] represents the aligned Λ vortices and has been determined experimentally. No theoretical model has been able to reproduce this type of breakdown. El-Hardy's breakdown (non-resonant[62] and resonant[59]) occurs in compressible supersonic flows. Gaster's work[43,44] of successive approximation, leading to an impulse response technique, is essentially non-parallel and made weakly non-linear by further approximation. On the other hand, the PSE method[66,80] is fully non-linear. Compressible BLs are reported to have 2-D TS waves in the early part of the development and 3-D hairpin vortices occur downstream in staggered (commonly occurring with large amplitudes) or aligned pattern when growth rate is suppressed by external forcing implicitly (complaint coatings) or explicitly (suction, favourable pressure gradient, cooling, etc.). Also, compressible BLs have multiple modes of primary oscillations, called Mack modes, which occur at supersonic Mach numbers. A brief review of existing transition models on supersonic BLs has also been presented.

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