TARGET ACQUISITION, TRACKING, SPACECRAFT ATTITUDE CONTROL, AND VIBRATION SUPPRESSION WITH IPFM REACTION JET CONTROLLERS

Hari B. Hablani†
Rockwell International, Space Systems Division, Seal Beach, CA

ABSTRACT

This paper demonstrates that thrusters, when administered by integral pulse frequency modulation (IPFM) control system, can be used successfully for time/fuel optimal acquisition and precision tracking of moving targets, with low jitter (2 µrad) of the focal plane of flexible spacecraft. The paper first derives roll, pitch, and yaw position, rate and acceleration commands for tracking a moving object with a payload initially on the zenith side of the spacecraft. These commands are then made explicit for landmark tracking. Conditions of visibility of the target to the spacecraft in a given celestial setting are also formulated. A non-iterative, closed-form, single-axis, time/fuel optimal algorithm is developed to slew the spacecraft from one arbitrary state (attitude and rate) to another to acquire a moving target. The formulae for designing the IPFM controller for tracking a given acceleration command profile with a rigid spacecraft are summarized next. Spacecraft flexibility concerns such as mode selection on the basis of spontaneous impulse response, stable or unstable control-structure interaction due to large flexible to rigid inertia ratios or due to symmetric modes having a moment arm from the vehicle mass center, and the elimination of instability using minimum-rise-time low-pass filter are addressed in depth. Extensive numerical results are included which conclusively establish that the IPFM controllers are eminently suitable for precision pointing and tracking and, surprisingly, even for vibration suppression if the flexible to rigid inertia ratio is below unity.

1. Introduction

This paper presents a variety of successful applications of a reaction jet control system for flexible, precision pointing spacecraft. Such spacecraft are designed to acquire and track moving objects, but because of their large inertias the reaction wheels — commonly used torquers — are frequently found to be inadequate for that task, unless wheels of higher torque and momentum capacity or more of them are used or unless control wheel gyro is resorted to. Both these alternatives, however, escalate the spacecraft cost. On the other hand, for stationkeeping or for changing altitude (orbit radius) and orbit inclination, spacecraft are often equipped with thrusters. It appears therefore that if the same thrusters, perhaps with slight modifications, could be used for acquiring targets rapidly and tracking them within desired pointing and jitter requirements, that will be very cost-effective. This seems specially true if the acquisition and tracking tasks were originally unplanned for and ensnare a program manager late in the design phase. With this motivation, the paper presents a procedure for designing integral pulse frequency modulation (IPFM) reaction jet controller for acquiring and tracking moving objects with flexible spacecraft. The scope of the paper along with a brief review of the pertinent literature is given below.

Command Profiles for Tracking

Section 2 of the paper is concerned with the derivation of position, rate, and acceleration command profiles for tracking a moving object. These profiles can be developed using quaternions, as demonstrated by Strikwerda et al.1 for Hubble Space Telescope and by Breckenridge and Man2 for Galileo scan platform. But because of easy physical interpretation, we will instead develop position commands in terms of Euler angles, following Reference 3. In the present context, the commands of Ref. 3 are not suitable because a) here the payload boresight is initially along zenith whereas that in Ref. 3 is along nadir, and b) Reference 3 employs a roll-pitch Euler angle sequence which becomes singular at 90° pitch angle, and for the case at hand the pitch angle does cross 90° while acquiring a near-earth target; so here we instead use the more natural pitch-roll sequence — more natural because the first rotation, pitch, compensates naturally for the once-per-orbit rotation about the pitch axis without intervention from the next, roll, rotation. In addition, the commands furnished here are more lucid than those in Ref. 3 in that, for landmarks on the rotating earth, the general commands are simplified and their dependence on spacecraft orbit parameters, orbital motion, and landmark's motion due to the earth's rotation is rendered explicit by stripping away all unit vectors. Regarding the rate and acceleration commands these are inertial and expressed in the spacecraft-fixed, central principal frame. Furthermore, whether a target is above the horizon or below is examined in more detail than in Ref. 3.

Target Acquisition

The word target is used here generically: it may denote an inertially fixed orientation (say, to track a star), a landmark moving because of the earth's rotation, or a missile or booster. In the first case, the final angular rates of the spacecraft about its three axes are zero, leading to a rest-to-rest maneuver problem, whereas in the second and third case the final rates are not zero and indeed they change if the (unknown) slew time tf varies in an iterative algorithm. In any event, in order to acquire a target, time/fuel optimal position and rate command profiles are generally employed as reference trajectories for a closed-loop controller, because the position and rate errors are then better controlled compared to when the position command is merely a step and the rate command merely zero, and also because the fuel consumption is minimized and the angular rates do not exceed the desired limits. Notwithstanding, Wie, et al.4,5 have illustrated the usefulness of controllers for large angle maneuvers based on step commands only. But step commands alone usually lead to large rate errors which in turn drive the spacecraft to high rates and cause significant fuel consumption. The time/fuel optimal reference trajectories may be based on single-axis analysis, as proposed by Redding and Adams (1987)6 for NASA Space Shuttle, or the eigen-axis (Euler-axis) approach, as currently implemented on the Space Shuttle; see Ref. 6 for comparison of the two approaches. D'Amario and Stubbs(1979)7 analyzed the eigen-axis approach and illustrated it in conjunction with a gyroless closed-loop reaction jet control system for a rest-to-rest maneuver. Not surprisingly, the Eigen-axis attitude maneuver approach was also used for the Apollo
The development of time/fuel optimal reference trajectories for target acquisition becomes tedious, though, in the case of those moving objects for which the eigen-axis may not be parallel with one of the principal axes and if the eigen-axis is time-varying; see Ref. 7 and Ref. 9, the latter offering a time/fuel optimal Euler rotation for a rest-to-rest maneuver about an oblique axis.

With this background, the objective of Section 3 can be stated. Redding and Adams,\textsuperscript{6} determined reference position and rate commands for acquiring a target iteratively; that is, given initial and final attitude and rate and desired rate limit, the durations of acceleration, coasting and deceleration, and reference coating rates were determined iteratively (see Trajectory Solution Algorithm, Fig. 6, Ref. 6). In contrast, Section 3 develops single-axis reference trajectories in closed-form and thus shows that iterations are not required.

**Reaction Jet Controllers**

Broadly speaking, reaction jet controllers might be put in two categories: 1) phase plane controllers, such as one for NASA Space Shuttle, and 2) modulation controller — those that modulate pulsewidth or pulse frequency or both. We hasten to add that this classification is artificial in that a controller from either category could be placed in the other; nonetheless, it enables us to interweave fine contributions of the past. One of the early designs of a phase plane controller is by Dahl, et al (1961),\textsuperscript{10} who related the switching lines in the phase plane with the deadbands in position and rate signals. Furthermore, they studied the limit cycles and performance of this controller in the presence of both stabilizing and destabilizing aerodynamic and gravitational torques on the spacecraft, leading to elliptic or hyperbolic trajectories in the phase plane. Mendel (1970),\textsuperscript{11} devised analogous relationships for a reaction jet controller having a nonlinear relay with asymmetric dead zone and a time delay in thrusters in the presence of both stable and unstable aerodynamic derivatives. Apollo spacecraft\textsuperscript{8} also employed a phase plane controller, its horizontal switching lines forming drift channels for time/fuel optimal behavior, instead of slanted lines of Refs. 10 and 11. In addition, it also used antisymmetric "shelf" deadzones in the phase plane, possibly to entrap shallow or delayed phase plane trajectories, as explained by Sacket and Kirchway (1981),\textsuperscript{12} and Penchuk et al (1985),\textsuperscript{13} in the context of Space Shuttle on-orbit flight controller. The phase plane controller for Space Shuttle is probably the most utilitarian, for it not only includes the drift channels and shelf deadzones, it contains as well a moving, biased switch curve whose location in the hysteretic region is determined by the estimated disturbance acceleration, unknown apriori, and its switch lines near deadband are parabolic instead of slanted straight lines of Ref. 8, 10, and 11. White et al (1976),\textsuperscript{14} have shown that such phase plane controllers are equivalent to time/fuel optimal controllers.

Coming to modulation controllers, Vaeth (1965),\textsuperscript{15} compared three of them: fixed pulse generator, derived rate feedback controller, and pulse-width-pulse-frequency (PWPF) modulation controller. The first and third controllers estimate spacecraft rate by passing attitude through a lead-lag compensation; and the second controller, by integrating the commanded control torque. Vaeth\textsuperscript{15} concluded that for the first two controllers, the required value of hysteresis would be impractical if the situation demands small minimum pulsewidth and large rate gain. To circumvent this obstacle, Scott (1967),\textsuperscript{16} introduced a sawtooth-pulse-reset circuit in parallel with a dual time constant feedback lag circuit around the relay with deadband and hysteresis (Schmitt trigger). While the sawtooth-pulse-reset allows the designer to select desired minimum pulsewidth independent of hysteresis, the dual time constant is useful in the case of saturating attitude sensor. DeBra\textsuperscript{17} has developed static characteristics of these controllers. Pseudo-rate controllers have been used on many spacecraft; Communication Technology Satellite (a.k.a. Hermes)\textsuperscript{18} for one. For the reasons that will be apparent momentarily, this paper focuses on the integral pulse frequency modulation (IPFM) reaction jet controller. Farrenkopf et al (1963),\textsuperscript{19} has given a complete procedure for designing this control system, and has compared its performance with saturating proportional-plus-derivative controller and a relay controller having a deadband and hysteresis. Abdel-Rahman (1977),\textsuperscript{20} has further compared it with the dual time constant pseudorate controller (without sawtooth-pulse-reset circuit) of Refs. 16 and 18. The unique characteristics and the ensuing benefits of an IPFM controller are: It emits a thruster pulse of constant width whenever the integral of a linear combination of position and rate error exceeds certain threshold; the frequency of the firings is thereby modulated, keeping the pulsewidth constant. Reference 19 concludes that because of integration of the errors, the effect of sensor noise is mitigated considerably and the attitude control is much smoother than that by pseudorate controller. This is true also because of availability of the rate error. Furthermore, owing to these features, the IPFM controller is less debilitated by structural flexibility than a pseudorate controller is, even if the natural damping of the structure is low ($\zeta < 0.005$) and the sensor time lag is large (Ref. 20). For these reasons, we have selected IPFM controller for target acquisition and tracking, which require large angle multi-axis slews, and for controlling the spacecraft in the presence of disturbance torques. Section 4 of the paper outlines the determination of the control parameters for these tasks.

**Reaction Jets and Spacecraft Flexibility**

Thrusters are particularly prone to exciting flexible modes of the spacecraft. So it is imperative to examine if flexiblility will degrade the control performance, induce instability, or under some favorable circumstances, associated vibrations will be suppressed by the controller (this turned out to be the case for IPFM controllers, see Sec.6). To examine this interaction, we begin in Section 5 with scalar metrics that identify important modes from among "infinite" of them [see Craig and Su (1990),\textsuperscript{21} for a review of model reduction practices]. The interaction between nonlinear reaction jet controllers and structures has been analyzed in the past using Describing Function as well as Liapunov technique; see, for example, Refs. 18, 22, 23 for the former and Ref. 20 for the latter. These analyses tend to be rather sophisticated; so, by way of contrast, we furnish in Section 5 a simple but intuitive criterion of stability involving the ratio of moment of inertia of the flexible and the rigid portions, $I_f/I_r$, of the spacecraft. Next, we observe that symmetric elastic modes usually do not interact with spacecraft attitude, but when they do, due to their moment arm from the vehicle mass center, thrusters must be located
carefully because their translational force is not of the same genre as the angle measured by an IMU gyro; the corresponding linear stability conditions for thrusters are derived in Section 5, following Gevarter (1970). Moreover, even if this linear stability condition is satisfied, unstable control-structure interaction becomes inevitable when a certain axis of the spacecraft is very flexible \( (I_f / I_r \gg 1) \) unless special filters are incorporated in the controller. Wie and Plescia, for example, demonstrated this for pulse-width-pulse-frequency modulation reaction jet controller for \( I_f / I_r = 5.8 \), and subsequently removed it by using a notch filter. The authors did not state the inertia ratio, but knowing the appendage frequency \( \Omega_1 \) and the vehicle mode frequency \( \omega_1 \), the ratio \( I_f / I_r \) is calculated from \( \Omega_1 / \omega_1 = (I_f / I_r + 1)^{-0.5} \). Because of sensitivity of the performance of the notch filter to modal parameters, a minimum-rise-time low pass filter is considered in Section 5 as an alternative. The filter owes its name to its attribute that its step response rises to unity in minimum time and, therefore, its phase lag is also minimum [Jess and Schussler (1965)]. Finally, Section 6 of the paper illustrates the analyses of Sections 2-5. Among other things, we illustrate that if the inertia ratio \( I_f / I_r \) is sufficiently small (20 ms), the IPFM controller indeed suppresses the structural oscillations, reinforcing the conclusions of Ref. 20. The paper is concluded in Section 7.

2. Position, Rate, and Acceleration Command Profiles for Tracking

Position Commands

Fig. 1 illustrates relative locations of the earth, spacecraft, and a target to be tracked. The spacecraft's orbit and the target's trajectory are both taken to be known. Measured from the earth's mass center, the spacecraft is at a location \( \mathbf{x}_s \) and the target at \( \mathbf{x}_t \), as shown in Fig. 1. The ideal line-of-sight (LOS) vector \( \mathbf{e} \), then, will be

\[
\mathbf{e} = -\mathbf{x}_s + \mathbf{x}_t
\]  

(1)

Ideally, this vector must be along the payload boresight, which in the case at hand is along the negative yaw axis \( ( -\mathbf{b}_3 \) unit vector, \( b \) for body). The remaining two spacecraft-attached unit vectors, \( \mathbf{b}_1 \) and \( \mathbf{b}_2 \), are along the roll and pitch axis, respectively. Before acquiring the target, the triad \( (\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3) \) is initially aligned with the local-vertical-local-horizontal unit vector triad \( (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) \) (\( e \) for circular orbit), implying that, initially, the payload boresight is along \( -\mathbf{e}_3 \) (looking zenith, see Fig. 1). Because the vectors \( \mathbf{e}_s \) and \( \mathbf{e}_t \) are known in geocentric inertial frame, the LOS vector \( \mathbf{e} \) can be calculated, after a chain of coordinate transformations, in the orbital triad \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \) expressed as

\[
\mathbf{e} = \mathbf{e}_1 \cdot \mathbf{e}_1 + \mathbf{e}_2 \cdot \mathbf{e}_2 + \mathbf{e}_3 \cdot \mathbf{e}_3
\]  

(2)

In order to bring the target to the focal plane center of the payload, the spacecraft is rotated about the vector \( \mathbf{e}_2 \) by an angle \( \theta_{yc} \) and then about the displaced \( \mathbf{e}_1 \) axis by an angle \( \theta_{xc} \). In the \( \mathbf{b}_1 \)-frame, the vector \( \mathbf{e} \) thus becomes

\[
\mathbf{e} = -\mathbf{e}_1 \mathbf{b}_3 \quad \mathbf{e} \Delta \mathbf{e}_2 \mathbf{e}_3
\]  

(3)

To determine \( \theta_{xc} \) and \( \theta_{yc} \), we express \( \mathbf{b}_3 \) in the orbital triad:

\begin{equation}
\mathbf{b}_3 = \cos \theta_{xc} \cos \theta_{yc} \mathbf{e}_1 - \cos \theta_{xc} \sin \theta_{yc} \mathbf{e}_2 + \sin \theta_{xc} \mathbf{e}_3
\end{equation}

(4)

where symbols \( \sin \theta = \sin \theta \) and \( \cos \theta = \cos \theta \) are used. Substituting Eq. (4) in Eq. (3) and comparing with Eq. (2) yields the desired commands:

\begin{align}
\theta_{yc} &= \tan^{-1} \left( -\mathbf{e}_1 \cdot \mathbf{e}_1 / \mathbf{e}_2 \cdot \mathbf{e}_3 \right) \quad (5a) \\
\theta_{xc} &= \sin^{-1} \left( -\mathbf{e}_1 \cdot \mathbf{e}_3 / \mathbf{e}_2 \cdot \mathbf{e}_3 \right) \quad (5b)
\end{align}

The negative signs in the numerator and denominator in Eq. (5a) are derived so as to identify the correct quadrant of the pitch angle \( \theta_{yc} \); meanwhile, the roll angle \( \theta_{xc} \) will always be \( -\pi/2 \leq \theta_{xc} \leq \pi/2 \). The commanded yaw angle, \( \theta_{zc} \), is zero and so is its rate \( \dot{\theta}_{zc} \); this, however, does not mean that the yaw component \( \omega_{zc} \) of the commanded inertial rate \( \omega_c \) of the spacecraft is zero. Exact expression of \( \omega_{zc} \) as well as the commanded roll and pitch rate components \( \omega_{xc}, \omega_{yc} \) of \( \omega_c \) will be derived shortly. The command equations (5) can be made more transparent if the target's motion is specified. For instance, if the target is a landmark at a longitude \( \lambda_T \) and latitude \( \phi_T \), and if the earth's radius is denoted \( R_E \) and its rotational rate \( \omega_0 \), the target vector \( \mathbf{x}_t \) then in the standard geocentric inertial frame will be

\[
\mathbf{x}_t = R_E \left[ c, \omega_a (\omega_0 t + \lambda_T), c, \omega_a s (\omega_0 t + \lambda_T), s \omega_a \right]^T
\]  

(6)

where the Greenwich hour angle between Vernal Equinox and Greenwich meridian is accommodated in the angle \( \omega_0 t \). To transform \( \mathbf{x}_t \) to the orbit frame \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \), denote the ascending node angle of the spacecraft orbit as \( \Omega_AN \) and the spacecraft true anomaly \( \omega_0 t \) measured from the ascending node line. \( \omega_0 \) is spacecraft orbit rate, and the two is in \( \omega_0 t \) and \( \omega_0 t \) usually differ by a constant.) Furthermore, the spacecraft location vector \( \mathbf{x}_s \) in the orbit frame is simply

\[
\mathbf{x}_s = -R_s \mathbf{e}_3 \quad \text{where} \ R_s \text{ is the spacecraft orbit radius. Using Eq. (1), the components of } \mathbf{e} \text{ in Eq. (2) can be determined. To lend lucidity to the command equations, we observe that, from space, the earth is seen as a disc of angular radius } \rho \text{ where (Wertz and Larson, Fig. 5.12)}
\]  

\[
\sin \rho = R_E / R_S
\]  

(7)

The general pitch command \( \theta_{yc} \), Eq. (5a), then reduces to this explicit form for landmark tracking.
where an auxiliary command angle $\theta_{xc}'$ is introduced to separate the influence of the spacecraft's once-per-orbit rotation, and it is governed by

$$\tan \theta_{yc}'(t) = \frac{c \cdot \mathbf{L}_a \cdot \mathbf{c} \cdot \mathbf{s} \cdot \mathbf{L}_a \cdot \mathbf{c} \cdot \mathbf{s} \cdot \mathbf{L}_a \cdot \mathbf{ci}}{c \cdot \mathbf{L}_a \cdot \mathbf{c} \cdot \mathbf{s} \cdot \mathbf{L}_a \cdot \mathbf{c} \cdot \mathbf{s} \cdot \mathbf{L}_a \cdot \mathbf{ci} - c \cdot \mathbf{L}_a \cdot \mathbf{c} \cdot \mathbf{s} \cdot \mathbf{L}_a \cdot \mathbf{ci}}$$

which can be transformed readily from the geocentric inertial frame to the orbit frame $\mathbf{L}_1, \mathbf{L}_2, \mathbf{L}_3$. Knowing the position command angles from the preceding, $\mathbf{\dot{L}}$ can then be expressed in the desired orientation of the body frame $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ using the pitch-roll sequence. Next, according to Eq. (3), the allowable change in $\mathbf{\dot{L}}$ in the body frame is

$$\mathbf{\dot{L}}^{\prime} = -\mathbf{\dot{L}}^0 \cdot \mathbf{b}_3$$

where an overcircle denotes differentiation in a rotating frame. Now, following Ref. 3, the inertial roll and pitch rate commands, $\omega_{xc}$ and $\omega_{yc}$, respectively, are found to be

$$\omega_{xc} = \mathbf{\dot{L}}^0 \cdot \mathbf{b}_2 / \mathbf{\dot{L}} \quad \omega_{yc} = -\mathbf{\dot{L}}^0 \cdot \mathbf{b}_1 / \mathbf{\dot{L}}$$

(11a,b)

Alongside, we also discover that the unknown rate of change $\mathbf{\dot{L}}^0$ is indeed

$$\mathbf{\dot{L}}^0 = -\mathbf{\dot{L}}^0 \cdot \mathbf{b}_3$$

(12)

Next, in order to determine the yaw component $\omega_{zc}$ of the commanded inertial rate $\omega_c$, we recognize that, in all, the spacecraft has three desired angular rates: clockwise once-per-orbit rotation $-\omega_0 \mathbf{b}_2$, pitch rate command $\dot{\theta}_{yc} \mathbf{b}_2$, and roll rate command $\dot{\theta}_{xc} \mathbf{b}_1$; the commanded yaw angle $\theta_{zc}$ and the rate $\ddot{\theta}_{zc}$ are both zero because they are not required for target tracking. The total commanded rate $\mathbf{\omega}_c = \omega_{xc} \mathbf{b}_1 + \omega_{yc} \mathbf{b}_2 + \omega_{zc} \mathbf{b}_3$ is thus

$$\mathbf{\dot{\omega}}_c = [\omega_{xc} \quad \omega_{yc} \quad \omega_{zc}]^T$$

(13)

Clearly, $\omega_{yc}$, known from Eq. (11b), is related to $\omega_{zc}$ as

$$\omega_{zc} = -\omega_{yc} \tan \theta_{xc}$$

(14)

which, surprisingly, does not reduce to a second-order quantity even when the roll angle $\theta_{xc}$ is small.
In Reference 3, the visibility condition is given in yet another but equivalent form involving the angle \( \delta \) between the LOS vector \( \ell \) and the unit vector \( \ell_3 \):

\[
\text{for visibility: } \quad \ell \leq R_S \cos \delta \quad (16c)
\]

In Fig. 2, all three conditions (16) are satisfied for the landmark \( T_1 \) and violated for the landmark \( T_2 \).

Consider now a moving target such as a missile or a booster (\( T_3, T_4, T_5 \) in Fig. 2). The target \( T_3 \) obviously satisfies the above three conditions and is clearly visible to the spacecraft. In contrast, the target \( T_5 \) is below the horizon and its invisibility is confirmed also by the conditions (16). But, although the target \( T_4 \) is above the horizon and clearly visible to the spacecraft, the conditions (16) are violated. In this circumstance — a moving target above the horizon — the conditions (16) are not useful and, instead, the following condition should be checked:

\[
\text{for visibility when } \{ \begin{array}{l}
R_S \leq R_S \sin \delta \\
or \rho \leq \delta
\end{array} \quad (17)
\]

where \( \rho \) is defined via Eq. (7).

3. Target Acquisition

Time/Fuel Optimal Single-Axis Command Profile from One Arbitrary State to Another

An ideal target acquisition scenario is depicted in Fig. 3 wherein the target position and rate profiles are those derived in the last section and the spacecraft’s profiles are time/fuel optimal. In order to acquire the target the spacecraft is slewed from its initial, known attitude \( \theta_i \) and rate \( \omega_i \) to its final desired position \( \theta_f \) and rate \( \omega_f \) at time \( t = t_f \). Clearly, \( \theta_f \) and \( \omega_f \) depend on \( t_f \) and at \( t = t_f \) the target profiles coalesce with spacecraft’s profile as shown in Fig. 3. Typically, the angles \( \theta_i \) and \( \theta_f \) are Euler angles and \( \omega_i, \omega_f \) the inertial rates in a body-fixed frame; but in this section we assume that \( \omega_i \) and \( \omega_f \) are the same as the Euler rates \( \dot{\theta}_i \) and \( \dot{\theta}_f \). This assumption is valid for uncoupled single-axis motion but not for large angle multi-axis motion. Although the large angle acquisition problem is usually multi-axis, only single-axis problem is considered below. Now, while the pair \( \theta_i, \omega_i \) is available from an IMU gyro, the final pair \( \theta_f, \omega_f \) is obtained from the target’s position and rate command profiles constructed in the previous section. These profiles predict when the target of interest will become visible and what the LOS attitude and rate will be at that instant in order to track it. The quantities \( \theta_i, \omega_i, \theta_f, \omega_f, \omega_L, \alpha_{mx} \) are all shown positive in Fig. 3 but they are not necessarily so in a real scenario; here \( \omega_L \) is the signed coasting rate limit, and \( \alpha_{mx} \) is the maximum acceleration provided by the thrusters, times a factor \( Q \) (say, 0.8) to ensure some control margin for compensating the lag in a closed-loop controller. Regarding the sign of the first torque pulse, it may be intuitively evident that

\[
\text{sgn } \alpha_{mx} = \text{sgn } (\theta_f - \theta_i) \quad (18a)
\]

This equation is not true for all possible combinations of \( (\theta_i, \omega_i) \) and \( (\theta_f, \omega_f) \); for instance, if \( (\theta_f - \theta_i) \) is zero, we then instead have

\[
\text{sgn } \alpha_{mx} = \text{sgn } (\omega_f - \omega_i) \quad (18b)
\]

But for the intended applications of this analysis — large angle maneuvers for target acquisition — Eq. (18a) is true because the angles \( (\theta_f - \theta_i) \) is usually large (say, one or two radians) whereas the rates \( \omega_i, \omega_f \) are usually very small (mrad/s). The following scheme to determine the periods \( t_1, t_L \), and \( t_2 \) (see Fig. 3 for the definition of these symbols) is based on simple single-axis dynamics, so to conserve space the details of the derivation are not provided.

We first assume that the magnitude \( l_0 \), of the intermediate peak rate \( \omega_L \) will be less than the allowed maximum slew rate \( l_0 L \), and calculate tentative \( t_1 \) and \( \omega_L \) as follows. Define

\[
\theta_{m} = \frac{\Delta (\theta_f - \theta_i) + (\omega_f^2 - \omega_i^2) / 2 \alpha_{mx}}{\alpha_{mx}} \quad (19)
\]

It can then be shown that the time duration \( t_1 \) (or \( t_L \)) in Fig. 3 is given by

\[
t_1 = \left\{ -\omega_i \pm \sqrt{\omega_i^2 + \alpha_{mx} \theta_m} \right\} / \alpha_{mx} \quad (20)
\]

Of the two values of \( t_1 \), if they have opposite signs, the positive root is selected, and if they are both positive, the smaller root is selected. In the numerical example considered in the Numerical Results and Discussion, the two roots have opposite signs. Knowing a unique value of \( t_1 \) (or \( t_L \)) thus, the rate \( \omega_1 \) is calculated from

\[
\omega_1 = \omega_i + \alpha_{mx} t_1 \quad (21)
\]

If \( l_0 \) < \( l_0 L \), the tentative values of \( t_1 \) and \( \omega_1 \) are accepted; otherwise, we introduce a coasting phase, with the sign of \( \omega_L \) set to be that of \( \omega_1 \), and \( l_0 L \) decreased to \( l_0 L \) such as

\[
\text{sgn } \omega_L = \text{sgn } (\omega_i + \alpha_{mx} t_1) \quad (22a)
\]

\[
\omega_1 = \omega_L \quad (22b)
\]

Fig. 3. Single-axis time/fuel optimal position, rate, and acceleration command profiles for target acquisition
and \( t_1 \) given above by Eq. (20) is now replaced with
\[
t_1 = \frac{(\omega_L - \omega_2)}{\alpha_{mx}} \tag{23}
\]
The coast interval \( \tau_L \) is calculated from
\[
\omega_L \tau_L = (\theta_f - \theta_i) + \left(\frac{\omega_1^2 + \omega_2^2}{2} - \omega_L^2\right) / \alpha_{mx} \tag{24}
\]
and the deceleration period \( \tau_2 \) is obtained from
\[
\tau_2 = \tau_1 - \frac{(\omega_f - \omega_1)}{\alpha_{mx}} \tag{25}
\]
When coasting is not required, \( \tau_L = 0 \), and \( \tau_2 \) is still furnished by Eq. (25), with \( t_1 \) dictated by Eq. (20) instead of (23). The total slew time, \( t_f \), is thus
\[
t_f = t_1 + \tau_L + \tau_2 \tag{26}
\]
Having determined \( t_1 \) and \( t_2 \), it is elementary to generate the reference commands \( \theta(t) \) and \( \omega(t) \). Once the time \( t_f \) has elapsed, these time/fuel optimal reference commands and the acquisition controller are taken over by the tracking commands developed in Section 2 and a tracking controller.

4. Integral Pulse Frequency Modulation (IPFM) Reaction Jet Controller

Modus Operandi

A single-axis IPFM reaction jet controller is shown in Fig. 4. As explained below, it is a modified version of the one considered in Refs. 19, 20 and 27. In this last reference, Bernussou formulates a varying integral threshold, denoted \( A_1 \) in Fig. 4, so as to keep the attitude error extrema nearly the same whether external disturbances are present or not; \( A_1 \) is constant in this study, nonetheless. The unique features of the IPFM controller in Fig. 4 are: time-varying position and rate commands \( \Theta_c \) and \( \Theta_c^* \), a rate gyro to measure spacecraft rate \( \dot{\theta} \), and a minimum-rise-time lowpass filter (Section 5) to strain out excited flexible modes from the noisy gyro signal \( \theta_n \) (Fig. 4) if the axis under consideration is overly flexible. When the IPFM controller is used to slew the spacecraft or acquire a particular target, the command \( \Theta_c \) and \( \Theta_c^* \) will be the ideal time/fuel optimal trajectories developed in Section 3; on the other hand, for tracking a target, the position and rate commands of Section 2 will be used. For simple attitude stabilization, both \( \Theta_c \) and \( \Theta_c^* \) will be zero. An IMU gyro such as Kearfott SKIRU IV (a very precise and quiet gyro) is included in Fig. 4 instead of an attitude sensor and lead-lag compensation (as in Refs. 19, 20, 27) because precise rate and attitude measurements facilitate better control of the attitude and jitter caused by elastic modes, as demonstrated in Section 6. Very briefly, the IPFM controller functions as follows.

The spacecraft attitude \( \theta \) and the rate \( \dot{\theta} \) comprise rigid and elastic modes, as shown in Fig. 4. The rigid attitude is denoted \( \Theta \) and the attitude angle contributed by \( v \)-th elastic vehicle mode, \( \theta_V \), where \( \theta_V(t) = \Phi_V \eta_V(t) \), and \( \Phi_V \) equals the \( v \)-th rotational modal coefficient at the sensor location and \( \eta_V(t) \) is the \( v \)-th modal coordinate. Clearly,
\[
\theta(t) = \Theta(t) + \sum_{v=1}^{\infty} \theta_V(t) = \sum_{v=1}^{\infty} \Phi_V \eta_V(t) \tag{27}
\]
The attitude rate counterpart of Eq. (27) passes through the IMU gyro of bandwidth \( \omega_g \), giving rise to the noise-free output \( \dot{\theta} \); when that is blended with gyro noise \( \dot{\theta}_g \), it becomes the noisy attitude rate \( \dot{\theta}_n \) which is then passed through, if necessary, a minimum-rise-time lowpass filter, yielding thereby \( \dot{\theta}_nf \), a

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4. Integral Pulse Frequency Modulation (IPFM) Reaction Jet Controller

**Fig. 4. Single-axis integral pulse frequency modulation reaction jet controller for a flexible spacecraft**
quantity very similar but not identical to the rigid rate \( \dot{\theta} \). The filter is of minimum-rise-time type, so the phase lag introduced by it is minimum, or equivalently, the output \( \dot{\theta}_nf \) rises to track the ideal output \( \dot{\theta} \) in minimum time. When the axis under consideration is moderately flexible, the low pass filter is not used, and \( \dot{\theta}_nf = \dot{\theta} \). Because a rate integrating gyro yields incremental attitude of the spacecraft, the attitude \( \theta_n \) or \( \theta_{nf} \) can be obtained from, respectively, \( \dot{\theta}_n \) or \( \dot{\theta}_{nf} \), as shown for \( \theta_{nf} \) in Fig. 4. Knowing the command profiles \( \Theta_c \) and \( \dot{\Theta}_c \), the position and rate errors are calculated as shown, and a composite error signal \( e \)

\[
e = \theta_e + K_D \dot{\theta}_e \tag{28}
\]

is formed and its integral \( e_I \) (\( e_I = e \)) is compared with the threshold \( \pm A_1 \) at a desired sampling frequency (50 Hz, perhaps). When \( |e_I| \geq A_1 \), the thrusters are fired for a pulsewidth \( \tau_w \) (\( \tau_w = 20 \text{ ms, say} \)), and the polarity of the control torque \( T_c \) is determined by

\[
\text{sgn} \ T_c = \text{sgn} \ e \tag{29}
\]

When appropriate thrusters are turned on, the integral \( e_I \) is reset to zero and the error signal \( e \) is integrated afresh (with the initial condition \( e_I = 0 \)). The integration proceeds even while the thrusters are on, to ensure that, if necessary, they will be saturated; that is, for sufficiently large error signals or disturbance torques, there will be no time gap between two consecutive, minimum pulsewidth firings. The control torque \( T_c \) and the disturbance torque \( T_d \), if the latter is present, act on the spacecraft, producing the attitude rate \( \dot{\theta} \), and the control system retraces the above operation in a closed-loop fashion.

**Determination of Control Parameters for Rigid Spacecraft: Single-Axis**

A reaction jet controller is required to counteract significant disturbances such as those caused by stationkeeping or orbit adjustment forces, producing torques if the resultant force does not pass through the vehicle mass center; see Wie and Plescia\(^22\), for example, for designing a pulse-width-pulse-frequency modulator in this circumstance. Analogously, when a spacecraft is slewed or commanded to track a target, the controller confronts inertial resistance of the spacecraft equal to the central, principle moment of inertia times the desired slew or track acceleration. The controller then must prevail against the extreme inertial resistance and slew the spacecraft or track a target within desired tracking accuracy. With these situations in mind, below we outline the design of an IPFM controller, given the maximum expected disturbance or resistance torque \( T_d \). Subsequently, we will comment on how to design this controller for just attitude control, with no disturbances to act against.

**A) In the Presence of a Constant Disturbance**

Under a constant disturbance torque \( T_d \), a reaction jet controller exhibits a limit cycle behavior shown in Fig. 5a (for \( T_d > 0 \) and control torque \( T_c < 0 \)). The steady state attitude offset \( \Theta_{SS} \) is the specified, tolerable attitude error in the presence of \( T_d \). To determine the integral threshold \( A_1 \), denote the vehicle's central principal moment of inertia about the axis under consideration as \( Iv \), and minimum angular rate increment due to the control torque \( T_c \) as \( \mu \). Then, using simple single-axis dynamics and the equilibrium condition between the control and disturbance torques, one can show that (Refs. 19, 27)

\[
A_1 = \Theta_{SS} \mu / T_d \tag{30a}
\]

where \( \alpha_d = |T_d| / I_v \)

\[
\mu = |T_c| \tau_w / I_v \tag{30b,c}
\]

and \( \tau_w \) equals the minimum pulsewidth of the thrusters. The two natural error extrema shown in Fig 5a can also be proved with ease. Ideal in Fig. 5a implies that the control torque fires instantaneously, imparting the angular momentum \( T_c \tau_w \) to the spacecraft momentarily. The period \( T_{ld} \) of this limit cycle in the presence of the constant disturbance torque \( T_d \) is given by

\[
T_{ld} = \mu / \alpha_d \tag{30d}
\]

Next, the rate gain \( K_D \) is decided according to the desired equivalent viscous damping coefficient, denoted \( \zeta_c \), of the controller. Specifically, following Reference 19, define a parameter \( K \)

![Fig. 5. Limit cycles and static saturation characteristics of IPFM controller](image)
which is related to the damping coefficient \( \zeta_c \) thus:

\[
K = 1 / (4 \zeta_c^2)
\]

(32)

Now, if \( \zeta_c \) is specified to be 0.707, then \( K = 0.5 \). Knowing \( K \), \( A_1 \) and \( \mu \), the rate gain \( K_D \) is determined from Eq. (31). The frequency \( \omega_c \) of the equivalent linear controller is then given by

\[
\omega_c = \sqrt{2 / K_D}
\]

(33)

Note that the linear equivalence is valid only during transient phase and not during limit cycle operation, whether in the presence of a disturbance torque or otherwise. The symmetric limit cycle parameters (Fig. 5b) in the absence of a disturbance torque, are denoted \( \Theta_b \) and \( \Theta_L \). The limit cycle rate \( \dot{\Theta}_L \) will be,

\[
\dot{\Theta}_L = \mu/2
\]

(34)

while the limit cycle attitude \( \Theta_L \) equals

\[
\Theta_L = A_1 / (2K_D)
\]

(35)

Accordingly, the symmetric limit cycle period \( \tau_L \) will be given by

\[
\tau_L = 4 \Theta_L / \dot{\Theta}_L
\]

(36)

Finally, static saturation limits in attitude and attitude rate are reviewed in Fig. 5c (Ref. 19). When the errors fall outside the zone covered by the two characteristics in Fig. 5c, the errors saturate. Notice that the steady-state attitude offset \( \Theta_{SS} = A_1 / \tau_{LD} \) and limit cycle attitude \( \Theta_L = A_1 / \tau_{LD} \) are within the saturating error limits \( \pm A_1 / \tau_w \) because \( \tau_{LD} > \tau_w \). Clark and Franklin (1969)\( ^2 \) have examined these saturation characteristics in great detail.

### B) No Disturbance

In this case, the limit cycle rate \( \dot{\Theta}_L \) is still calculated using Eqs. (30c) and (34), but now the limit cycle attitude \( \Theta_L \) is specified according to the desired performance. The limit cycle rate \( \tau_L \) is then determined using Eq. (36). The rate gain \( K_D \) is arrived at next using the relationship

\[
K_D = \tau_L / 2
\]

(37)

Knowing \( \Theta_L \) and \( K_D \), the integral threshold \( A_1 \) is finally obtained from Eq. (35).

### IPFM Controller for Multi-Axis Tracking

Fig. 6 depicts the three-axis IPFM controller for flexible spacecraft for target tracking, not acquisition. A significant difference between this multi-axis controller and the single-axis controller in Fig. 4 is that here the small angular tracking errors \( \theta_x \) and \( \theta_y \) in roll (x-axis) and pitch (y-axis) are obtained directly from the off-centered position \( (\xi_x, \xi_y) \) of the target image on the x-y focal plane. The line-of-sight vector \( \xi \) defined by Eq. (1) is denoted here \( \xi_L \), \( \xi_C \) and \( \xi_B \) to indicate the inertial, the local-vertical-local-horizontal and the body frame, respectively, and \( \xi_B = [\xi_x \xi_y \xi_z]^T \). The matrices \( \xi_C \) and \( \xi_B \) in Fig. 6 are the inertial-to-orbital and orbital-to-body frame transformation matrices. In the simulation, focal plane noise is added to the noise-free position errors \( \theta_x, \theta_y \) which are then quantized and differentiated to obtain the inertial rate errors \( \omega_x, \omega_y \) in the body frame. These rate errors could be obtained from the gyros also but the procedure just mentioned is more direct, provided the focal plane errors \( \xi_x, \xi_y \) are measurable in flight. Besides, during target tracking, the star trackers might not be able to continue tracking stars, so the gyro drift errors might be overwhelmingly large. Regarding yaw axis, because the corresponding position and rate errors cannot be obtained from the focal plane errors, they are derived from a rate gyro, as shown before for single-axis IPFM controller. The determination of control torque commands for three axes proceeds as before. Another difference between single-axis and three-axis controllers in Fig. 4 and Fig. 6 is that, for precision tracking, the deviation \( \delta_0 \) of the spacecraft motion from a circular orbit caused by the thruster force vector \( \mathbf{F} \) (or any other source for that matter) is considered now. Excitation of the elastic modes due to the force \( \mathbf{F} \) and the torque \( \mathbf{T} \) is also modeled but care must be exercised in not accounting for this excitation twice through the force \( \mathbf{F} \) as well as the torque \( \mathbf{T} \) of thrusters. Also, note that while \( \delta_0 \) is the deviation of the spacecraft mass center from the circular orbit, \( \delta_0 \) (actually, \( \delta_0 \) plus circular orbit radius and other unmodeled deviations) is what will be obtained from the inertial navigation system located at a certain node in the bus. Finally, the rate commands \( \omega_c \) in Fig. 6 are those developed in Section 2 for tracking a target.

For large angle multi-axis acquisition of the target, the position and rate commands of Section 2 provide the spacecraft state at the end of the acquisition phase, and the spacecraft is slewed to this final state from its arbitrary initial state using time/fuel optimal multi-axis command profiles; these multi-axis profiles are not developed here, but Refs. 1, 2, 7 and 9 may be helpful in this regard.

### 5. Spacecraft Flexibility Considerations

The above IPFM controller design is for a rigid spacecraft, but it is intended to be used on a flexible spacecraft. So the concerns regarding mode selection, stable or unstable control-structure interaction, and avoidance of instability using a filter are addressed below.

#### Mode Selection

Let the frequency of mode \( \nu \) be denoted \( \omega_\nu \), and the associated damping coefficient, \( \zeta_\nu \). The attitude angle \( \theta_\nu (t) \) contributed by the \( \nu \)-th mode and introduced in Eq. (27), is then given by

\[
\dot{\theta}_\nu + 2\zeta_\nu \omega_\nu \dot{\theta}_\nu + \omega_\nu^2 \theta_\nu = \Phi_\nu \Phi_\nu \mathbf{e}_\nu \mathbf{T} (t)
\]

(38)

where \( T \) is the thruster's control torque about the axis under consideration and \( \Phi_\nu \mathbf{e}_\nu \) is an equivalent modal slope (defined below) at the thruster location. If the vehicle mode under consideration is antisymmetric and if the thrusters residing in the spacecraft bus produce a pure couple \( T_c \) at the vehicle mass center, \( \Phi_\nu \mathbf{e}_\nu \) is then the usual modal slope \( \Phi_\nu \) at the thruster location. Otherwise, \( \Phi_\nu \mathbf{e}_\nu \) is related to the translational modal coefficients at the thruster locations in a way that is given later in this section. For now, note that if the torque \( T(t) \) is treated as an impulse of angular momentum \( T_c \tau_w \) (an appropriate representation of the thrusters)

\[
T(t) = T_c \tau_w \delta(t)
\]

(39)
with δ(t) as the Dirac delta, Hughes (1981)\textsuperscript{29} has then shown that for lightly damped modes, ζ_ν → 0, and for T_c τ_w equals unity momentum:

\[
\sqrt{\int_0^\infty \theta_\nu^2(t) \, dt} = | \Phi_νΦ_ν,eq| / \left( 2\sqrt{\zeta_ν \nu_ν^3} \right) \tag{40a}
\]

The metric (40a) is equivalent to the metrics depicting other circumstances formulated by Skelton et al (1982)\textsuperscript{30} and other investigators using controllability and observability grammans (Ref. 21). Next, we observe that impulses have uniform power spectral densities, and if, instead of an impulse, there exists a disturbance or a control torque T(t) having the spectral density P(ω), then Ref. 31 has shown that

\[
\sqrt{\int_0^\infty \theta_\nu^2(t) \, dt} = | \Phi_νΦ_ν,eq| / \left( 2\sqrt{\zeta_ν \nu_ν^3} \right) \tag{40b}
\]

where P(ω_ν) is the spectral density P(ω) at ω = ω_ν.

Selection of the modes based on (40a) or (40b) is appropriate when interest centers around long-term steady-state behavior of modes. But when the analyst is concerned with immediate effects of thruster firings on a mode, like we are, the impulse response of a mode is more relevant. It is elementary to show that the change in the amplitude of the modal rate ̇θ_ν(t), denoted Δ̇θ_ν, due to the impulse T_c τ_w is
\[ \Delta \theta_v = \Phi_v \Phi_{v,eq} T_c \tau_w \quad \text{(41)} \]

and, consequently, the change in the amplitude of \( \theta_v(t) \) will be

\[ \Delta \theta_v = \Phi_v \Phi_{v,eq} T_c \tau_w / \omega_v \quad \text{(42a)} \]

Assuming \( T_c, \tau_w \) to be unity, the modes can be selected on the basis of (42a) instead of (40a) or (40b). Not surprisingly, \( \Delta \theta_v \) is independent of damping coefficient \( C_T \) because now we focus on a quarter or half modal period immediately after the impulse and in this duration damping has no significant influence on the modal response. For completeness, we note that if a step torque \( T_c \) is the source of modal excitation, one may then use the maximum amplitude of \( \theta_v(t) \), apparent in half modal period, as the selection criterion:

\[ \theta_{v,max} = 2 \Phi_v \Phi_{v,eq} T_c / \omega_v^2 \quad \text{(42b)} \]

Of course, the factor 2 \( T_c \) in Eq. (42b), being the same for all modes, could be dropped while comparing mutual significance of the modes.

**Influence of Modes on Reaction Jet Controller Operation**

Describing function and Liapunov technique apart, it seems that for a reaction jet control system to be effective on a flexible spacecraft, the changes \( \Delta \theta_v \), Eq. (41), should be

\[ \Delta \hat{\theta}_v << 2 \hat{\theta}_L \quad \text{(43)} \]

The condition (43) will generally be satisfied if the ratio of the moment of inertia \( I_f \) of the flexible parts to the moment of inertia \( I_r \) of the rigid parts, both determined at the vehicle mass center and about the axis under consideration, is much less than unity:

\[ I_f / I_r << 1 \quad \text{(44)} \]

To substantiate this, assume for now that \( \Phi_{v,eq} = \Phi_v \) (that is, assume that only antisymmetric modes contribute to the attitude motion; special conditions under which this is not true is considered below). Then it is known that

\[ \sum_{v=1}^{n} I_v \Phi_v^2 = I_f / I_r \quad I_v = I_f + I_r \quad \text{(45)} \]

Moreover, according to Eq. (30c) and Eq. (34)

\[ I_v \hat{\theta}_L = \frac{1}{2} T_c \tau_w \quad \text{(46)} \]

Therefore, using (41) and the assumption \( \Phi_{v,eq} = \Phi_v \), we obtain the ratio

\[ \Delta \hat{\theta}_v / (2 \hat{\theta}_L) \sim I_v \Phi_v^2 \quad \text{(47)} \]

The condition (43) is thus equivalent to

\[ I_v \Phi_v^2 << 1 \quad \text{(48)} \]

For a spacecraft consisting of a central rigid body and flexible appendages, the modes can be arranged such that

\[ \Phi_1^2 > \Phi_2^2 > \Phi_3^2 > ... \quad \text{(49)} \]

where the modes 1, 2, 3, ... are those that interact with the axis under consideration. By virtue of (45) and (49) then, the condition (48) is loosely equivalent to the condition (44).

The condition (43) considers steady state limit cycle regime. In order to avoid a harmful, excessive excitation of elastic modes during transients, while the reaction jets are away from limit cycling, the equivalent linear controller frequency \( \omega_c \) should be a decade smaller than a critical modal frequency \( \omega_v \):

\[ \omega_c << \omega_v \quad \text{(50)} \]

To sum up, the elasticity of a spacecraft is unimportant if the conditions (44) and (50) are satisfied. If not, the more they are encroached upon, the more important the elastic modes become. And the important modes are then retained according to the metrics \( \Delta \hat{\theta}_v \) and \( \Delta \theta_v \) [Eq. (41) and Eq. (42a)] \( v = 1, 2, 3, ... \) with \( T_c \tau_w \) equal unity.

**Instability Due to Symmetric Elastic Modes: Linear Analysis**

When the mass distribution of a spacecraft is symmetric, the attitude motion of its bus interacts with antisymmetric vehicle elastic modes only. However, for the generic spacecraft shown in Fig. 7, because of its moment arm from the vehicle mass center to the composite mass center of the two solar arrays, y-axis interacts with symmetric vehicle elastic modes—mode 1, for instance. This complicates the selection of the thrusters in that, now, any arbitrary pair that produce y-torque will not be acceptable lest it produce instability, despite the colocation of the thrusters and the gyros residing in the central rigid body. This issue is resolved below.

An IPFM (perhaps others as well) reaction jet control system is equivalent to a linear proportional-plus-derivative controller. It is well-known that this linear control system does not destabilize a vehicle flexible mode if the actuator and the sensor are colocated and they are both of the same genre: a torque actuator accompanies an attitude sensor, and a force actuator accompanies a linear displacement sensor; see Conclusion 3 of Gevarter (1970)\(^2\). If this requirement is not satisfied, the colocation does not guarantee the stability of flexible modes; Gevarter\(^2\), for instance, shows that when a control force is colocated with an attitude sensor at a certain interior station of a beam-like spacecraft, the first (symmetric) flexible mode is unstable, whereas if they are both located at one of the free ends, the same mode is stable. [Figure 3, Ref. 24].

Mathematically, when a rectilinear force acts on a flexible mode of a spacecraft, the mode is governed by [cf. Eq. (38)]

![Fig. 7. Generic deformable spacecraft with two solar arrays](image-url)
\[ \ddot{\theta}_v + 2\zeta_v \omega_v \dot{\theta}_v + \omega_v^2 \theta_v = \Phi_v \sum_j \chi_{vj}^T I_j \]

(\(v = 1, \ldots, \infty\))  \hspace{1cm} (51)

where \(I_j\) is the force vector produced by the \(j\)-jet and \(\sum_j\) extends over just those thrusters that are fired simultaneously to produce torque about a certain axis. The vector \(I_j\) may be rewritten in terms of the jet's direction cosine vector \(a_j\), which may be different from a unit vector because of possible thruster canting:

\[ I_j = a_j f \]  \hspace{1cm} (52)

where \(f\) is the scalar force produced by each \(j\)-jet assuming that all the jets, firing simultaneously, produce the same thrust \(f\). Now, assume that these jets produce a control torque \(T_c\) at the vehicle mass center about a certain axis; then, if \(\mathcal{L}_{eq}\) denotes the equivalent moment arm, the torque \(T_c\) will be given by

\[ T_c = f \mathcal{L}_{eq} \]  \hspace{1cm} (53)

With the aid of Eqs. (52) and (53), Eq. (51) transforms to

\[ \ddot{\theta}_v + 2\zeta_v \omega_v \dot{\theta}_v + \omega_v^2 \theta_v = \Phi_v \left\{ \sum_j \chi_{vj}^T a_j \right\} \mathcal{L}_{eq} / \mathcal{L}_{eq} \]  \hspace{1cm} (54)

Therefore, the stability of the mode \(v\) requires (Gevarter 24)

\[ \Phi_v \left\{ \sum_j \chi_{vj}^T a_j \right\} / \mathcal{L}_{eq} > 0 \]  \hspace{1cm} (55)

which is different from the following usual requirement of stability of a mode for a pair of colocated actuator and sensor:

\[ \Phi_v^2 > 0 \]  \hspace{1cm} (56)

This difference arises because the condition (56) is valid for antisymmetric modes only, whereas (55) is valid for any mode; the former can be deduced from the latter though, as follows. For any mode, it is known that the translational modal displacement \(X_{vj}\) at a point \(I_j\) from the reference origin but still within the central rigid body, can be written as

\[ X_{vj} = X_{v0} - I_j \times \Phi_v \]  \hspace{1cm} (57)

where \(X_{v0}\) is the modal displacement of the origin and \(\Phi_v\) the modal rotation vector of the rigid body; \(I_j \times \) is the \(3 \times 3\) skew symmetric matrix associated with the vector \(I_j\) and equivalent to the vector cross product. Substituting (57) in (51), and introducing the relationship

\[ I_j = L_c + \frac{P_j}{r} \]  \hspace{1cm} (58)

where \(L_c\) is the vector from the reference origin to the vehicle mass center and \(P_j\) is the vector from the vehicle mass center to the \(j\)-th thruster location, we arrive at

\[ \ddot{\theta}_v + 2\zeta_v \omega_v \dot{\theta}_v + \omega_v^2 \theta_v = \Phi_v \left\{ \sum_j \chi_{vj}^T I_j + \left( \Phi_v \right)^T P_j \times I_j \right\} \]  \hspace{1cm} (59)

where \(\chi_{v0}\) is the translation of the point that represents the vehicle mass center and it is given by:

\[ \chi_{v0} = X_{v0} - L_c \times \Phi_v \]  \hspace{1cm} (60)

We emphasize that \(\chi_{v0}\) is not the translation of the vehicle mass center itself because, by definition, no linear, or angular momentum for that matter, resides in a vehicle elastic mode. Specializing (59) to a scalar torque case, we obtain:

\[ \ddot{\theta}_v + 2\zeta_v \omega_v \dot{\theta}_v + \omega_v^2 \theta_v = \Phi_v \sum_j \chi_{vj}^T a_j f + \Phi_v^2 T_c \]  \hspace{1cm} (61)

Because for an antisymmetric mode the modal translation of the vehicle mass center is, by definition, zero, the condition (56) follows from (61) for stability of such modes, whereas, for a symmetric mode, \(\chi_{v0} \neq 0\), and therefore more general condition (55) or its equivalent

\[ \Phi_v \left( \sum_j a_j \right) / \mathcal{L}_{eq} + \Phi_v^2 > 0 \]  \hspace{1cm} (62)

must be called upon. In conclusion, for stability, among the jet pairs that control the rigid mode, those be selected that satisfy the most the inequality (62) for a critical mode, so that the jets suppress the elastic mode as well (provided the ratio \(I_f / I_r\) is favorable).

**Avoidance of Instability About Very Flexible Axis: Minimum-Rise-Time Lowpass Filter**

When the condition (43) or, equivalently, condition (44) is violated, the thruster firing causes excessive oscillations of at least one elastic mode, leading to spacecraft instability. In that instance, as stated in the Introduction, minimum-rise-time lowpass filter can be used beneficially. The transfer function \(H_{LP}(s)\) where \(s\) is the Laplace variable, of one such filter used in this paper is

\[ H_{LP}(s) = \frac{\sigma_f \omega_{f1}^2 \omega_{f2}^2}{s^2 + \omega_z^2} \times \frac{2}{(s + \sigma_f) \prod_{i=1}^{4} (s^2 + 2 Z_{fi} \omega_{fi} s + \omega_{fi}^2)} \]  \hspace{1cm} (63)

where, in terms of the desired stopband frequency \(\omega_{fs}\), defined below, various parameters are:

\[ \sigma_f \triangleright \sum_f \omega_{fs} , \quad \omega_{f1} \triangleright \Omega_f , \quad \omega_{f2} , \quad \omega_{z} \triangleq \Omega_z \omega_{fs} \]  \hspace{1cm} (64)

and, according to Ref. 25,

\[ \Omega_f = 0.631 \quad \Omega_{f2} = 0.85 \quad \Omega_z = 1.08215 \]  \hspace{1cm} (65)

\[ Z_{f1} = 0.815 \quad Z_{f2} = 0.473 \quad \sum_f = 0.548142 \]

In \(H_{LP}(s)\), note the presence of a pair of imaginary zeros \(\pm j \omega_z\) \((j^2 = -1)\) as in the case of a notch filter. Because of the last definition in (64) and \(\omega_z\) is very close to the stopband frequency \(\omega_{fs}\).To determine \(\omega_{fs}\) we note that if \(\omega_z\) equals the troublesome modal frequency \(\omega_v\), these oscillations will be filtered out completely, and so the stopband frequency \(\omega_{fs}\) may be set to

\[ \omega_{fs} = \omega_v / \Omega_z \]  \hspace{1cm} (66)

Usually the modal frequency \(\omega_v\) is not known with precision, so Eq. (66) may be altered to

\[ \omega_{fs} = \sigma \omega_v / \Omega_z \]  \hspace{1cm} (67)

where \(\sigma\) is the level of confidence on the NASTRAN-predicted modal frequency \(\omega_v\). Due to the numerator \(s^2 + \omega_z^2\) in the transfer function (63), the second derivative of the input to the
filter is required. Because this input is noisy attitude rate signal (Fig. 4), a third-order digital fading memory filter [Zarchan (1990)] may be employed to obtain double-derivative of the input with desired smoothness.

6. Numerical Results and Discussion

The analysis and control schemes discussed in the preceding sections were implemented on two spacecraft — one shown in Fig. 7 and the other with only one solar array. Using nonlinear digital simulation, controller performance was studied under varied circumstances; due to space limitations, only the most revealing results are presented below.

Command Profiles

Fig. 8, illustrates segments of ground tracks of a spacecraft in a circular orbit and a target in a certain trajectory. The objective is to command a payload, attached rigidly on the zenith side of the spacecraft, to track the target. The direction of the LOS-vector from spacecraft to the target varies with time as displayed by the succession of arrows in Fig. 8, with time t as parameter. It is clear from Fig. 8 that the target is out of the spacecraft orbit plane, so both in-track (pitch, \( \theta_{pc} \)) and cross-track (roll, \( \theta_{sc} \)) commands are required for tracking the target.

The commands, in the pitch-role sequence, for tracking the launchsite of the target are shown in Fig. 9a, and the corresponding inertial rate commands about the three body axes in Fig. 9b. (These are not the commands for tracking the target in Fig. 8.) Fig. 9a demonstrates that, in the present case, the launchsite acquisition involves large slew angles and nonzero body rates at the end of acquisition. Although the body rates shown in Fig. 9b for tracking a landmark are small (<6 mrad/s), they are usually much different (larger) while tracking a flying object. Interestingly, from the moment the cross-track command angle (\( \theta_{xc} \)) exceeds 45°, the magnitude of the yaw rate (\( \omega_{zc} \)) becomes greater than that of the pitch rate \( \omega_{yc} \) [refer to Eq. (14)], although the commanded yaw angle and yaw Euler rate are still zero.

Time/Fuel Optimal Single-Axis Launchsite Acquisition: Rigid Spacecraft

Knowing the initial and the desired position and rate of the spacecraft in an acquisition scenario, the time/fuel optimal command profiles are obtained according to the algorithm developed in Section 3. These profiles are then used to drive an acquisition IPFM controller whose parameters are determined for the given control torque as explained in Section 4. The performance of one such controller for acquiring a launchsite in pitch axis (y-axis) is shown in Fig. 10a through Fig. 10e. The initial spacecraft rate \( \omega_{1} \), its final desired rate \( \omega_{f} \), and the specified coast rate \( \omega_{L} \) are stated in Fig. 10a, while the initial pitch angle is zero and the final pitch angle at \( t_{f} = 173.89 \) s is -125°. In Fig. 10a we observe that the actual position and rate of the spacecraft follow the command profiles very closely, with little overshoots. The position error \( \theta_{e} \) equal to \( \theta_{yc} - \theta_{y} \), not detectable in Fig 10a, and the rate error \( \dot{\theta}_{e} \) are displayed in Fig. 10b. The controller is designed for a steady state error \( \Theta_{SS} \) equal to 0.25 deg (4.363 mrad), and therefore the position errors during acceleration and deceleration phases rise to a value slightly greater than 0.25 deg. The sampling frequency of the IPFM controller is 50 Hz. If this were larger (say, 100 Hz), \( \theta_{e} \) would settle almost exactly on 0.25°. It must be pointed out that the error \( I_{e} \) would remain settled at ~5 mrad for as long as the acceleration and deceleration phases last (presently, \( t_{1} = 9.188 \) s and \( t_{2} = 7.668 \) s, Fig. 3) because the IPFM-controlled spacecraft behaves as a mass-spring-damper system with \( \zeta_{c} = 0.707 \) and \( \omega_{c} = 0.55 \) rad/s [Eq.(33)], and because the acceleration command is equivalent to a constant disturbance torque. As each phase ends, the position error recedes to the double-sided limit cycle behavior, seen vividly in Fig. 10c. Notice that the limit cycle amplitude \( \Theta_{L} \) is approximately one-hundredth of the steady-state offset error \( \Theta_{SS} = 5 \) mrad.

The graceful time response of the IPFM controller (for a rigid spacecraft, though) in Figs. 10a - 10c is in much contrast with the pitch response of the PWPF (pulse-width pulse-frequency) controller under similar circumstances in Fig. 8, Ref. 22. The spacecraft in Ref. 22 although flexible, is relatively rigid in pitch axis because the structural pole and zero, 5.61 and 5.53 rad/s, respectively, are nearly the same; the comparison therefore is fair. As a side comment, the unattractive features of the PWPF response are 1) significant oscillations of the position error around \( \Theta_{SS} \) in the presence of a constant disturbance, and 2) limit cycle amplitude when the disturbance is zero as much as the steady state error \( \Theta_{SS} \). These features arise because the PWPF torque command, unlike that of the IPFM, is decided by the instantaneous position and rate errors and not by their integrals. Moving on to Fig. 10d, it exhibits position and rate errors in the phase plane, along with the saturation characteristics of the controller. At \( t=0 \), both errors are zero, and after a brief delay, the controller responds, accelerating the spacecraft. To keep pace with the time/fuel optimal rate command, the controller saturates temporarily, bringing the rate error down, and then it edges along the saturation line, near \(-\Theta_{sat} \) attitude error. When the acceleration phase in the command profiles is over, the controller pulls in gently the position error towards the origin where it performs limit cycles (not visible in Fig. 10d). Finally, when the deceleration phase begins, the above sequence replays itself antisymmetrically. The associated control torque history along with the time period between two successive firings and fuel consumption are shown in Fig. 10e. These agree entirely with Fig. 10a - 10d.

Usually, it is the moving target that is to be acquired, not its launchsite. Although the target acquisition will be qualitatively the same as the above launchsite acquisition, the former will not be illustrated here.

Single-Axis Landmark Tracking: Rigid Spacecraft

After acquisition, the tracking controller takes over which is usually different and more precise than the acquisition controller, and is designed according to the specified maximum track-acceleration and tolerable track-error. Fig. 11a shows a pitch acceleration profile for tracking a landmark for the entire time its visible to the spacecraft. The maximum track-acceleration, 0.014 deg/s² (0.25 mrad/s²), occurs at B and C locations and the acceleration is virtually zero when the landmark is near horizon at A's. The IPFM controller for this acceleration profile is designed for \( \Theta_{SS} = 75.0 \) μrad (compared to ~5 mrad earlier for the acquisition controller) and a maximum track-acceleration of 1 mrad/s². The corresponding four one-sided limit cycles for ±1 and ±0.25 mrad/s² acceleration are...
**Fig. 8.** Ground tracks of a spacecraft and a target on the rotating earth

**Fig. 9a.** Sequence pitch-roll: Euler angle commands $\theta_{xc}$ (roll) and $\theta_{yc}$ (pitch) for tracking a launchsite

**Fig. 9b.** A comparison of commanded body rates $\omega_{xc}$, $\omega_{yc}$, and $\omega_{zc}$ for tracking a launchsite

**Fig. 10a.** Launchsite acquisition about pitch axis with IPFM controller using time/fuel optimal position and rate commands: Actual and commanded position and rate profiles
Fig. 10b. Position and rate errors during acquisition

Fig. 10c. Limit cycle behavior while coasting

Fig. 10d. Phase Plane: Rate Error versus Position Error During Acquisition

Fig. 10e. Control torque history, time period $t_k$ between two successive firings of IPFM controller, and fuel consumption
ASSUMPTIONS: A) Rigid Spacecraft; B) Impulsive Firings, C) Analog Controller

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**Fig. 11.** Steady state limit cycles associated with different track-accelerations

shown in Fig. 11b; shown also is the two-sided symmetric limit cycle corresponding to the "A" locations. Note that the position error jitter (extrema) of the limit cycles shrink as \( \Theta_{SS} \) or the track acceleration escalates. These limit cycles are steady-state responses of the IPFM controller to constant track-accelerations. But when the track-acceleration varies, as in Fig. 11a, the A limit cycle shifts gradually to B, to C, and then to A again. Fig. 12a demonstrates interesting pleated staircase pattern of tracking a slowly varying roll-rate command (-5 \( \mu \text{rad/s}^2 \) acceleration) with an IPFM controller, starting with zero position and rate error at \( t=0 \). Fig. 12b shows the associated, quasi-sawtooth rate error profile, varying essentially between \( \pm \Theta_{SS} \). Fig. 12c depicts the position error in tracking. Compared to the A symmetric limit cycle in Fig. 11, Fig. 12d shows the actual asymmetric limit cycles within almost \( \pm 2\Theta_{L} \) in the phase plane formed with the position and rate errors of Figs. 12b and 12c.

**Multi-Axis Target Tracking: Flexible Spacecraft with One Solar Array**

These results are obtained using the multi-axis controller presented in Fig. 6. Fig. 13 shows the roll, pitch, and yaw errors of a spacecraft having one solar array while tracking a flying object. This scenario was shown earlier in Fig. 8 and the corresponding position and rate commands are similar to, but more vigorous than, those portrayed in Figs. 9a and 9b. To design the IPFM controllers, the limit cycle attitudes \( \Theta_{L} \) about the three axes are specified to be 2.5, 2.5, and 100 \( \mu \text{rad} \) which render roll and pitch controller much tighter than the yaw controller. From the acceleration command profiles [Eqs. (15)], the extreme roll and pitch accelerations are found to be 1.3 \( \text{mr/s}^2 \), and the extreme yaw acceleration is assumed to be one-twentieth of that (Eq. (15c) was not then available). The remaining control parameters are calculated as described in Section 4. Regarding the vehicle flexibility, the solar array's transverse bending (0.47 Hz, fundamental vehicle mode) induces roll, while torsion and in-plane bending (1.38 and 1.77 Hz, fundamental vehicle modes) induce pitch and yaw motions. The incremental attitude rates due to the minimum impulse bit of the thrusters, Eq. (41), correspond to the three modes, satisfy, though differently, the intuitive stability criterion (43). Although the z-axis is found to have the least margin, that is compensated by keeping a loose z-controller because the yaw pointing accuracy is not critical.

Returning to Fig. 13, the position errors, on the average, are seen to change in proportion with the track-acceleration (not shown here), for \( \Theta_{SS} = \Theta_{L} \alpha_d / \mu \) [Eq. (30a)] and the parameters \( \Theta_{L} \) and \( \mu \) are constant. Actual extrema of the roll and pitch errors are slightly greater than the predicted extrema (-61.5 \( \mu \text{rad} \)) because of finite (50 Hz) sampling frequency and flexibility. The limit cycles vary with the disturbance acceleration \( \alpha_d \) as
Fig. 12a,b,c. An IPFM controller tracking a roll rate command with nearly 5 μrad/s² acceleration.

Fig. 12d. Phase plane behavior corresponding to Fig. 11b and c.

Fig. 13. Roll, pitch, and yaw errors while tracking a moving object with IPFM controllers.

depicted earlier in Fig. 11, with the amplitude of jitter diminishing as $a_d$ increases. The actual extreme $\Theta_{SS}$ for yaw-axis ($-0.0021$ rad) near $t = 135$ s is twice the design value. After this, large yaw errors (0.0125 rad) develop because the yaw track-acceleration changes rapidly (along with the roll and yaw track-accelerations) but the yaw controller is intentionally unexacting. Fig. 14 illustrates the control torque history about the three axes. When the roll and pitch track accelerations are much smaller than the control acceleration, double-sided firings take place; these are gradually replaced by one-sided more frequent firings as the desired accelerations increase.

The most eventful, yet benign, control-structure interaction is seen in the roll-axis, interacting with the transverse bending modes. While the intuitive criterion (43) considers just one pulse, the cumulative effect of the pulses and track-acceleration on the flexible modes in terms of their contribution to the roll angle and rate (denoted $\theta_{xf}$ and $\dot{\theta}_{xf}$) is displayed in Fig. 15. Most of $\theta_{xf}$ arises from the fundamental mode at 0.47 Hz. As the track-acceleration gradually rises, at a rate much slower than 0.47 Hz, the solar array bends quasi-statically, with a maximum contribution of nearly 50 μrad to the roll angle. The cumulative
flex-roll rate $\dot{\theta}_{xf}$ usually remains within $\pm \dot{\Theta}_L$. Equivalent, open-loop viscous damping coefficient of all modes in this study is taken to be 0.0025.

**Vibration Suppression: Flexible Spacecraft with Two Arrays**

Consider now a rest-to-rest x-slew of the spacecraft shown in Fig. 7 with an IPFM controller driven by time/fuel optimal position and rate command profiles. Relevant control and modal parameters are summarized in Table 1. The spacecraft slew is much similar to the acquisition process illustrated in Fig. 10a - 10e, so the slew itself is not illustrated to conserve space. But Fig. 16 illustrates the excitation and suppression of flex-roll angle $\theta_{xf}$ and rate $\dot{\theta}_{xf}$ contributed by all important modes. In an open-loop system, during the first acceleration pulse, the elastic modes oscillate from zero to twice the static deformation. But here we observe that the IPFM controller stunts the growth of the oscillation from the beginning itself and quickly suppresses it to its static deformation level (it cannot be suppressed any further, with the thrusters residing in the spacecraft bus) and very small rates. During coasting, the spacecraft does not accelerate, so the new static deformation is zero and the IPFM controller promptly suppresses the earlier static deformation to nearly zero amplitude. (Actual magnitudes of the elastic contributions in Fig. 16 and the following figures are not important.) During deceleration phase and subsequently, the controller re-displays its vibration suppression characteristic. We must point out that the coasting phase in the slew process not only reduces fuel consumption, it affords as well the controller an interval to suppress the rigid offset error in the slew angle of the spacecraft (0.25 deg, Fig. 10b) and the positive static deformation $\theta_{xf}$ in Fig. 16; otherwise, the immediately following deceleration pulse will suddenly impose opposite rigid offset error and opposite static deformation, accentuating prior errors and usually destabilizing the spacecraft.

![Fig. 14. Roll, pitch, and yaw control torque history for tracking](image)

![Fig. 15. Contribution of ten flexible modes to the roll attitude and rate](image)

![Fig. 16. Excitation and suppression of elastic mode 4 (0.97 Hz) during 90° slew](image)
Vibration suppression with the IPFM controller is further illustrated in Fig. 17, related with the control of the spacecraft under a disturbance torque arising from velocity-correction thrust for the first 20 seconds. Because this axis interacts with a symmetric transverse bending mode (see Table 1), the thruster pair is carefully selected so as to satisfy the basic, linear stability condition (55). In the first 20 s, the IPFM controller counteracts the disturbance torque, so the average net torque on the spacecraft is zero. The associated zero mean excitation of the symmetric transverse bending mode (0.28 Hz), expressed in terms of its contribution $\theta_y$ to the pitch angle, is shown in Fig. 17. Termination of the disturbance torque at $t = 20$ s spoils the equilibrium and thereby aggravates the vibrations, but the IPFM controller soon suppresses the elastic mode (Fig. 17), eliminating alongside 0.25° pitch offset error, not shown but similar to that in Fig. 10b. This pitch and earlier roll vibration suppression is in line with the satisfaction of the stability criterion Eq. (43) (Table 1). As for the yaw axis, the criterion (43) or (44) is not satisfied because $I_r/I_x = 1.7$ and, predictably, the $z$-IPFM controller designed by ignoring the flexibility is found to be unstable though simulation. This instability is presaged, though not uncovered, by the Bode plot of an equivalent linear proportional-plus-integral controller for $z$-axis shown in Fig. 18, wherein the gain corresponding to the vehicle mode 2 is nearly 7 (16.9 dB) and phase nearly 90°. This nonlinear instability may be apprehended by using the describing function technique of Ref. 23, but we do not attempt that here. To eliminate it, however, we employ the minimum-rise-time lowpass filter, Eqs. (63) - (65), whose gain and phase characteristics are shown in Fig. 19. The unity gain and zero phase of the filter in the low frequency regime and the notch characteristic and ~270° phase at the mode 2 frequency are evident in the figure. When the filtered attitude and rate are used in the controller, the corresponding Bode plot, with 10% error in the modal frequency, is shown in Fig. 20, where we observe that the gain at the mode 2 frequency is now nearly 0.15 (~16.5 dB) and the phase nearly 90°. When the $z$-IPFM controller with the filter is simulated in the presence of a constant disturbance, the controller not only stabilizes the $z$-axis, it even suppresses, albeit only slightly, the mode 2. This is illustrated in Fig. 21 where $\theta_{zf}$, the contribution of the mode 2 to the yaw attitude, exhibits damped oscillations about some static deformation for first 90 s while a constant disturbance torque is acting. When the disturbance torque is removed at $t = 90$ s, $\theta_{zf}$ oscillations shift to zero mean. The slow damping until $t = 163$ s is greater than that due to the simulated natural damping ($\zeta_v = 0.0025$) and is apparently caused by the thrusters. The sudden quenching of the mode at the end is due to a proportional-plus-integral-plus-derivative controller using reaction wheels and unfiltered thrusters, when operated by IPFM controllers, are viable alternatives to reaction wheels and control moment gyros for precision and low jitter acquisition and for tracking moving targets by flexible spacecraft. If the inertia ratios of the flexible portions of the spacecraft are fairly below unity, the IPFM controller suppresses even the vibrations of structural modes because of its unique attributes of integration of position and rate errors, constant pulsewidth, and modulation of thruster firing frequency.

7. CONCLUDING REMARKS

The preceding results conclusively establish that the integral pulse frequency modulation (IPFM) reaction jet controllers are useful for target acquisition and tracking as well as for spacecraft attitude control and vibration suppression. The problem has not been solved in its entirety yet; for instance, the topics such as multi-axis acquisition of a moving target (in which case the final attitude and rate of the spacecraft alter if the acquisition time alters), sensor noise, spacecraft and target navigation errors, and estimation of spacecraft attitude and rate and gyro bias and drift rate are yet to be considered. Nevertheless, it can be stated with confidence that even thrusters, when operated by IPFM controllers, are viable alternatives to reaction wheels and control moment gyros for precision and low jitter acquisition and for tracking moving targets by flexible spacecraft. If the inertia ratios of the flexible to rigid portions of the spacecraft are fairly below unity, the IPFM controller suppresses even the vibrations of structural modes because of its unique attributes of integration of position and rate errors, constant pulsewidth, and modulation of thruster firing frequency.

![Fig. 17. Pitch angle vibrations due to Mode 1 and 5 and their suppression with 15 lb Jets](image-url)
Fig. 18. Bode plot of an equivalent linear proportional-plus-derivative controller: z-axis 1 lb thruster and no filter.

Fig. 19. Bode plot of a minimum-rise-time lowpass filter for z-axis.

Fig. 20. Bode plot of an equivalent linear proportional-plus-derivative controller: z-axis 1 lb thrusters and minimum-rise-time lowpass filter, Mode-2 frequency 10% erroneous.

Fig. 21. Yaw Error due to mode 2 caused by velocity-correction maneuver, and its suppression with the jets and with the wheels at the end.

REFERENCES


17. DeBra, D.B., Pulse Modulators, Stanford University, Class Notes.


