Interplanetary Spacecraft Controllers Using Thrusters

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Design of attitude controllers for interplanetary spacecraft is addressed. The first controller is for Rhumb-line precession using an almost periodic train of thruster pulses at a constant phase with the sun. Earlier relationships are expanded and interlaced coherently and demonstrated on a Mercator plot. The second controller is for nutation damping. For spacecraft with products of inertia, a control scheme is formulated that involves four phase angles, 90 deg apart in a nutation cycle, where the positive or negative control impulses about the transverse axes are injected to damp the nutations. The third controller is for attitude control during aerobraking for planetary orbit insertion. The controller seeks and maintains a zero aerodynamic torque orientation. Because this orientation may correspond to a constant bias angle of attack measured from the instantaneous velocity vector turning at a nonuniform pitch rate, a pitch angular acceleration command profile is developed for parameterizing a thruster controller. The fourth, and last, controller is for soft landing. It comprises a longitudinal velocity controller enhanced by gravity turn and a constant velocity phase at low altitudes near ground, a lateral velocity controller producing commands to pitch spacecraft at varying rates, and a reaction jet controller for tracking the commanded pitch angle and rate until touchdown. Satisfactory performance of all controllers is amply demonstrated.

I. Introduction

This paper describes various controllers for interplanetary spacecraft such as Mars Environment Surveyor, Mars Global Surveyor (MGS), Mars Surveyor Program, and Mars Surface Exploration Pre-Project. The paper deals with both spin-stabilized and three-axis-stabilized spacecraft, and it consists of four main sections. Section II describes a technique for designing a Rhumb-line precession controller using an almost periodic train of thruster pulses at a constant phase with the sun. The relationships from different sources in the literature are expanded, coherently interwoven, and demonstrated on a Mercator plot. Because the precession of a momentum vector inevitably is accompanied by nutations, these high-frequency oscillations at the end of precession must be damped for satisfactory pointing accuracy of the spin axis.

A control scheme is devised and illustrated for this purpose in Sec. III. A spinning spacecraft with products of inertia exhibits constant biases in its transverse angular rates. To effect nutation damping of such spacecraft, a control scheme is taken up from the literature and homed. The scheme involves, inter alia, bandpass filters to eliminate biases from the transverse rates. Four phase angles, 90 deg apart in a nutation cycle, are determined analytically, where positive or negative control impulses about the transverse axes are fired for nutation damping. It is shown that when a spacecraft has small products of inertia, the final steady-state nutation angle equals the angle between the geometric spin axis and the adjacent principal axis.

Section IV presents an attitude controller, using thrusters, for the aerobraking phase of Mars orbit insertion. As a spacecraft approaches its destination planet, it is despun and its velocity is reduced to establish a planetary orbit. The apogee of this highly elliptic orbit is brought closer to the planet by aerobraking instead of using propellant for velocity reduction. For a portion of the orbit near periapsis, where aerobraking is effective, an attitude controller is designed to seek and maintain a zero aerodynamic torque orientation of the spacecraft. In the present application, this orientation corresponds to a constant, positive bias angle of attack measured from the instantaneous velocity vector. Because, in an elliptic orbit, this vector turns at a nonuniform pitch rate, a pitch angular acceleration command profile is developed and the parameters for integral pulse frequency modulation (IPFM) thruster controller are determined for tracking time-varying orientations.

Section V presents a soft-landing controller for the Mars Lander. It comprises 1) a longitudinal velocity controller employing a velocity-altitude reference trajectory for guidance, a gravity turn below 45 m, and a constant-velocity phase below 5-m altitude until a soft impact is sensed by the navigation system; 2) a lateral velocity controller that requires pitching the spacecraft first at a rate inversely proportional to the current slant range and then according to gravity turn activated at a suitable low altitude; and 3) an IPFM reaction jet controller for tracking the commanded pitch angle and rate profile until touchdown. A radar altimeter is used until the spacecraft descends to a 45-m altitude. Afterward, an inertial reference unit, inheriting the errors of the radar altimeter at 45 m, is used for landing. The optimum gains for both longitudinal and lateral controllers are determined, and first-order digital fading memory filters are used, the coefficients of which are optimized with stochastic response analysis to smooth noisy measurements.

Ample numerical results, within the space allowed, are presented to demonstrate effectiveness of all controllers. The relationship of the present work to the previous work is identified in each section below.

II. Rhumb-Line Precession of Spin-Stabilized Spacecraft

The spin axis of a spacecraft can be precessed along either a great arc or a Rhumb line between the initial orientation and the final orientation of a spacecraft in a celestial sphere. In the former case the phase angle of the thruster firing varies significantly with each spin cycle, leading to operational complexities, whereas in the latter case the phase angle is constant, easily controlled by a sun sensor. Rhumb-line precessions are therefore more common than great-arc precessions.

A. Analysis

Figure 1 depicts the geometry of spin-axis precession produced by thrusters. Let \( x_0, y_0, z_0 \) be the spacecraft-fixed frame, with \( z_0 \) as the spin axis of the spacecraft. The precession torque \( \vec{F}_p = (P = \text{force}, \ell = \text{moment arm}) \) is generated about an axis forming an angle \( \beta \) with the transverse axis \( x_0 \). The spin angle of the spacecraft is denoted \( \phi \) and is measured from the ascending node line \( x' \) as shown in Fig. 1. Let \( t_p \) denote the short pulsedwidth (20–200 ms) of the thrusters. The angular momentum imparted by each pulse is then \( F t_p \), and the precession angle \( d\phi \) about an axis transverse to both torque axis and spin axis is (see Fig. 1) \( d\phi = F t_p / (I \omega_{x_0}) \), where...
The total change in the colatitude angle $\theta$ is obtained by integrating the equation $d\theta = -dP \sin \phi_0$, yielding

$$d\theta = -P \cos \Phi \frac{\sin \phi_0}{\cos \Phi}$$

where $P$ is the total precession angle affected by $N$ firings arrived at by summing up all $dP$ for $N$ firings:

$$P = \frac{N \Phi \Omega}{Iwo}$$

Whereas colatitudes $\theta_1$ and $\theta_2$ are dictated by the mission and spacecraft–sun geometry, the phase angle $\Phi_1$ is calculated from Eq. (3) and $\Phi_N$ or $\Phi_2$ from $\Phi_N = \phi_t - \pi/2$, $\Phi_2 = \phi_t + \pi/2$; Eq. (4) then yields the required precession angle $P$, and Eq. (5) the required number $N$ of firings

$$N = \frac{\theta_2 - \theta_1}{\cos \Phi \frac{\Phi \Omega}{Iwo}}$$

where $\Phi = \Phi_1$ or $\Phi_2$. When the pulsewidth $t_p$ is not small (that is, the condition $\omega \cdot t_p/2 \ll 1$ is not satisfied), a more accurate version of Eq. (6), presented in the unabridged version of this paper, is used. Equation (6) must be integerized to its nearest integer to minimize deviation of the actual, final longitude and colatitude from the desired values $\psi_2$ and $\theta_2$. Moreover, to assess the fuel efficiency of a Rhumb-line precession, one may compare numerically the precession angle $P$, Eq. (5), with the great arc length $F$ between the initial and the final latitudes and longitudes

$$F = \cos^{-1}[\cos \psi_1 \cos \psi_2 + \sin \psi_1 \sin \psi_2 \cos (\psi_2 - \psi_1)]$$

As stated earlier, the phase angle $\phi_t$ is required to be the same for each thruster firing. Because the phase angle or, equivalently, $\theta_0$ ($\phi = 0$, $t = 0$), is measured from the ascending node line $x'$, whose orientation angle $\psi$ varies slowly with each thruster firing, the spin angle $\phi$ does not cross the value $\phi_t$ at a uniform spin-period interval. Consequently, a sun sensor must be used to ensure that the spin angle $\phi$ equals $\phi_t$ for each thruster firing. Finally, a comment about the nutations of the spacecraft while precessing: Typically, the spin period of a spacecraft is an irrational multiple of the nutation period and therefore the nutation amplitude does not grow monotonically with thruster firings; instead, it waxes and wanes and is self-limited.

### B. Illustration

Six first-order equations governing dynamics and inertial orientation of a spinning spacecraft are simulated. Table 1 in Ref. 3 furnishes the parameters used for simulating a Rhumb-line precession illustrated conceptually in Figs. 19.7 and 19.8 of Wertz. The control torque is produced by two $z$ thrusters located diametrically opposite on the rim of the spacecraft, firing oppositely. Figure 2 shows a dashed Rhumb line (not clearly visible) joining the initial and the final specified locations of the spin axis on a Mercator plot. Because the Rhumb-line length $P$ is 75.5 deg whereas the corresponding great circle arc $F$ is 59 deg, this Rhumb-line precession is inefficient compared with the great-circle precession. The advantage, nonetheless, of the Rhumb-line precession is the constancy of the phase angle $\phi_1$, equal to 31.71 deg in this example, within $\pm 0.25$-deg measurement accuracy of the sun sensor. Figure 2 illustrates, in addition, the undulations of the nutation loops accompanying the precession; their amplitudes do not grow unremittingly because the nutation period, 27.33 s, is not an integer multiple of the spin period, 20 s. The corresponding nutation angle of the spacecraft, denoted $\gamma$ and defined as

$$\gamma = \frac{(I_1 \omega_1)^2 + (I_2 \omega_2)^2}{I_3 \omega_3}$$

varies discontinuously with each firing, remaining constant in the interim. Numerical results show that the angle $\gamma$ is within 0.4 deg and does not increase. The longitude $\psi$ and the colatitude $\theta$ vary with time, starting from their initial values and reaching their final desired values at the end of precession in 5630 s, remaining essentially constant afterward—essentially because the spacecraft continues to
nutate with some residual amplitude after precession, indicating the need for active or passive nutation damping.

### III. Active Nutation Damping

This section is concerned with spacecraft with zero and $yz$ products of inertia and is based on Ref. 5. Although Ref. 5 assumes that the spacecraft spins about the minor axis, the following analysis is for the major spin axis. Hence, according to the definition of the body-fixed frame $x_i,y_i,z_i$ in Fig. 1, $I_3 > I_2 > I_1$.

#### A. Control Strategy

Consider the equations of motion governing the transverse rates $\omega_1$ and $\omega_2$, influenced by the unspecified control torques $N_1$ and $N_2$ for nutation damping. Following Devey et al., define quasinutation frequencies

\begin{align}
\omega_{q1} &= \frac{(I_3 - I_2)\omega_1}{I_1} \\
\omega_{q2} &= \frac{(I_3 - I_1)\omega_1}{I_2} \\
\omega_0 &= \sqrt{\omega_{q1}\omega_{q2}}
\end{align}

Assuming that the transverse rates $|\omega_j| \ll \omega_0$ ($j = 1, 2$), the spin speed $\omega_0$ remains essentially constant: $\omega_0 \approx \omega_0$. The free response, therefore, of the two equations of motion governing transverse angular rates can be written as

\[ \omega_1 + jk\omega_2 = \rho_0 \exp j(\theta_0 + \alpha) \]

where $j^2 = -1$, $k = \sqrt{\omega_{q1}/\omega_{q2}}$, $\alpha = \omega_0(t - t_0)$, and $\rho_0$ and $\theta_0$ are initial amplitude and phase of the motion $\omega_1(t)$ and $\omega_2(t)$ at $t = t_0$:

\begin{align}
\rho_0 &= \left(\omega_{q1}^2 + k^2\omega_{q2}^2\right)^{1/2} \\
\theta_0 &= \tan^{-1}(k\alpha_0, \omega_{q2})
\end{align}

Further, Ref. 3 shows that, for nutation damping with the control torque $N_1$ about the $x$ axis, the $x$-angular impulses imparted by the thrusters must be

\begin{align}
\Theta_{\theta_0} &= -\frac{\alpha}{2} : N_1 \Delta t = \frac{-k_2I_1\rho_0(\alpha/2)}{\sin(\alpha/2)} \\
\Theta_{\theta_0} &= \pi - \frac{\alpha}{2} : N_1 \Delta t = \frac{k_2I_1\rho_0(\alpha/2)}{\sin(\alpha/2)}
\end{align}

where $\Delta t = t - t_0$ is selected such that Eqs. (12) are satisfied for a small $k_2$; $k_2 = 0.05$, for example. For $y$ thrusters, the angular impulses for nutation damping are found to be

\begin{align}
\Theta_{\theta_0} &= \pi - \frac{\alpha}{2} : N_2 \Delta t = \frac{-k_2I_2\rho_0(\alpha/2)}{\sin(\alpha/2)} \\
\Theta_{\theta_0} &= \pi = \frac{\alpha}{2} : N_2 \Delta t = \frac{k_2I_2\rho_0(\alpha/2)}{\sin(\alpha/2)}
\end{align}

For detailed derivation, see Ref. 3.

#### B. Spacecraft with Product-of-Inertia $I_{yz}$

Here, we assume that only $I_{yz} = \int yz dm$ product of inertia is nonzero. Because the products of inertia are typically much smaller
than the moments of inertia, it is beneficial for analytical ease, to treat each product of inertia separately. The following analysis therefore can be retraced for $I_{23}$ and $I_{31}$ products of inertia individually, though we do not attempt that here. We also point out that the above definition of $I_{23}$ includes the negative sign, unlike the convention used by Devey et al.5

Proceeding with the vector equation of motion of a rigid body, with $I_{23}$ in the inertia matrix, three scalar equations are derived for zero external torque acting on the spacecraft. We then invoke typical and presently legitimate assumptions that $|a_j| < |a_k|$ for $j = 1, 2; |I_{23}| \ll (I_1, I_2, I_3)$; and $a_2 \approx a_3$, where $a_2$ is a constant spin speed. The transverse equation governing $\omega_2$ possesses a forcing term due to the product-of-inertia $I_{23}$. Denote this by $\mu$ and introduce a nondimensional product-of-inertia parameter $\nu$ as follows:

$$\mu \triangleq \frac{a_2^2 I_{23}}{I_1}$$  

$$f \triangleq \frac{I_{23}^2}{(I_2 I_3)}$$

The product of inertia alters the nutation frequency $\omega_n$, Eq. (9c), to

$$\omega_n^2 = \frac{\omega_n^3 (a_2 + f a_3)}{1 - f}$$

The transverse rates obtained by solving the transverse equations exhibit the following forced response:

$$\omega_1 = (\mu/\omega_n) \sin \omega_n t$$

$$\omega_2 = (\mu/\omega_n) (1 - \cos \omega_n t)$$

from which we conclude that the transverse rate $\omega_2$ after nutation damping, will settle on its bias value $\omega_{2b} = \mu/\omega_n$. As one would anticipate, $\omega_{2b}$ is related to the tilt angle $\lambda$, about the geometric axis $x_2$, between the orthogonal axes $y_2 z_2$ and the principal axes $y_2 z_2$. In particular, it can be shown that, for small tilt angles, $|\lambda| < 1$, the moments-of-inertia $I_1$ and $I_3$ are essentially the principal values and $I_{23} \approx (I_2 - I_3)\lambda$, which indicates that, in the present application, because $I_1 > I_2$, the product-of-inertia $I_{23}$ and the tilt angle $\lambda$ have opposite signs. With the aid of Eqs. (9a) and (14a), the bias rate $\omega_{2b}$ is found to be equal to $-\omega_0 \lambda\omega_1$, matching with Eq. (39) of Ref. 5 except for the negative sign arising from the difference that Ref. 5 deals with spin about the minimum moment of inertia and this paper is dealing with spin about the maximum moment-of-inertia.

Let $b_1, b_2, b_3$ be the unit vectors along the geometric, spacecraft-attached frame $x_1 y_1 z_1$. We then observe that the steady-state spin rate of the spacecraft, $\omega$, equals $b_1 \omega_1 + b_2 \omega_2 + b_3 \omega_3$ because, in the principal frame $x_p y_p z_p$ (with $p_1$ as the unit vector along the principal spin-axis $z_p$), in view of the relationship $\omega_2 = -\omega_0 \lambda$, the angular velocity $\omega$ equals $-\omega_0 \lambda p_2$, which induces no angular motion about the transverse principal axes $x_p$ and $y_p$. In the absence of nutation, the steady-state angular momentum $\mathbf{h}$ of the spacecraft in the $x_1 y_1 z_1$ frame is:

$$\mathbf{h} = (I_2 \omega_{2b} + I_3 \omega_{3b}) b_2 + (I_3 \omega_{3b} + I_2 \omega_{2b}) b_3$$

Substituting $\omega_{2b} = -\omega_0 \lambda, I_{23} \approx (I_2 - I_3)\lambda$ and ignoring the $\lambda^2$ term, $\mathbf{h}$ simplifies to

$$\mathbf{h} \approx -\lambda I_1 \omega_0 b_2 + I_3 \omega_0 b_3$$

The corresponding steady-state nutation angle $\gamma$, defined by Eq. (8), is then $\gamma = |\lambda|$. The steady-state values $\omega_{2b} = -\lambda \omega_0$ and $\gamma = |\lambda|$ help verify the numerical results.

To implement the nutation damping strategies (12) and (13), the phase angle $\theta_n$ must be determined. For the zero product-of-inertia case, this is accomplished using the definition (11b), wherein the unbiased transverse rates are sensed by gyros. A nonzero product of inertia, however, renders this procedure inadequate because gyros measure transverse rates containing biases. Therefore, the bias $\omega_{2b}$, determined above, and $\omega_{2b}$, if it exists due to the product-of-inertia $I_{23}$ not considered here, must be filtered out from $\omega_1$ and $\omega_2$ sensed by the gyros. This filtering is performed with a bandpass filter, one for each axis.

C. Bandpass Filters

The transfer function of a bandpass filter is

$$H_{BP}(s) = \frac{2 \zeta \omega_n s}{s^2 + 2 \zeta \omega_n s + \omega^2_n}$$

centered at the nutation frequency $\omega_n$; here, $s$ is the Laplace variable and $\zeta$ is a suitable damping factor. Because this filter is used as a part of a sampled control system, its following discrete form is derived:

$$H_{BP}(z) = \frac{b_0 - b_2 z^{-2}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

where $z$ is the discrete transform and the polynomial coefficients are defined in Ref. 3 in terms of the semisample angle $\delta_m = \omega_n t_s/2$, with $\omega_m = (2/\pi) \tan(\omega_n t_s/2)$ and $t_s$ equals the sample period. Let $\omega_{2k}$ denote the kth sample of the input angular rate $\omega_2$ at $t = t_k$, and $\omega_{2k} \delta_k$ denote the kth output sample. Furthermore, in the present application and typically, the angle $\delta_m$ is much less than unity and therefore $\delta_m^2 \gg 1$. Using this assumption, a simplified time-domain version of Eq. (20) is then

$$\omega_{2k+1} \delta_k = 2 \omega_{2k-1} \delta_{k-2} + \zeta \omega_{2k} (\omega_{2k} - \omega_{2k-2})$$

By using the discrete-time-domain version (21), we avert the need for the derivative $\omega_2$ of the angular rate $\omega_2$ in the continuous-time-domain version of Eq. (19). This is a significant benefit because the acceleration $\omega_2$ is not measured.

D. Numerical Results

For results concerning zero product of inertia, see Ref. 3. Figure 3 illustrates nutation damping when the product-of-inertia parameter $f$ is 0.01. The corresponding tilt angle $\lambda$ between the geometric spin-axis $z_2$ and the principal axis $z_2$ is $-10.3^\circ$ and the bias rate $\omega_{2b} = -\omega_0 \lambda$ equals 3.24 deg/s. The filtered rate $\omega_{2f}$ is obtained by using the time-domain discrete Eq. (21) with $\zeta$ equal to 0.2. The corresponding time delay for the filter output $\omega_{2f}$ to rise to its steady-state amplitude is nearly two nutation periods (54.7 s). Moreover, owing to the product-of-inertia $I_{23}$, the nutation angle $\gamma$, defined by

$$\gamma = \frac{(h_{11}^2 + h_{22}^2)^{\frac{1}{2}}}{h_{33}}$$

is not constant now. To damp the nutations, the control policy, Eq. (13), is invoked after $\omega_{2f}$ attains its steady state. As damping progresses, the nutation angle $\gamma$ (Fig. 4) approaches the magnitude of the tilt angle, $|\lambda| = 10.3^\circ$ deg. At the end of nutation damping,

**Fig. 3** Nutation damping in the plane $\omega_1-\omega_2$ with $\gamma$-torque thrusters.
IV. Thruster Attitude Control During Aerobraking

Real spacecraft are seldom symmetric and streamlined and therefore atmospheric drag generates a torque that must be nullified to keep the spacecraft stable. \(^1\) Aerodynamics of the aerobraking configuration of the spacecraft at hand reveals that the pitch torque coefficient—the only coefficient required in this study—is zero for a positive angle-of-attack \(\alpha_{ak,b}\) equal to 3.15 deg and, for aerodynamic stability or equivalently for positive pitch stiffness at angle-of-attack \(\alpha_{ak}\) equal to zero, the slope of this coefficient near the equilibrium angle \(\alpha_{ak,b}\) is negative, as desired. To null the aerodynamic torque acting on the spacecraft, \(\alpha_{ak}\) must be commanded to be equal to the equilibrium angle \(\alpha_{ak,b}\).

A. Analysis

Figure 5 depicts the geometric relationship between the angle-of-attack \(\alpha_{ak}\), pitch angle \(\alpha_2\), and the angle \(\xi\) between the velocity vector \(v\) and the local horizontal \(O_1\) in the orbit plane. The local vertical local horizontal orbit triad \(O_1, O_2, O_3\) and the spacecraft attached triad \(b_1, b_2, b_3\) shown in Fig. 5 are defined as usual. Because the nose of the spacecraft is along \(b_1\), the angle of attack is, by definition, \(\alpha_{ak} = -\alpha_2 - \xi\). The angle \(\xi\) varies with the spacecraft true anomaly \(\theta\) measured from the periapsis or with \(\eta\) measured from the line of ascending node (Fig. 5), and the objective of the pitch controller is to ensure that \(\alpha_{ak} = \alpha_{ak,b}\), that is, to keep the pitch angle \(\alpha_2\) equal to the commanded pitch angle \(\alpha_2c\), where \(\alpha_2c = \alpha_{ak,b} + \xi\), and \(\xi\) varying with the true anomaly \(\theta\) thus

\[
\tan \xi = \frac{e \sin \theta}{1 + e \cos \theta} \quad (23)
\]

where \(e\) is the eccentricity of the orbit.

To design a reaction jet controller for tracking a varying \(\alpha_2c\), corresponding inertial pitch acceleration command is determined, for it governs the minimum control torque required. The inertial pitch rate command \(\omega_2\), in terms of the pitch rate command \(\dot{\alpha}_2\), and the true anomaly rate \(\dot{\theta}\) is \(\alpha_2c = \alpha_{ak} - \xi = \dot{\theta} - \dot{\xi} - \dot{\theta}\). Differentiation of Eq. (23) then leads to

\[
\omega_2c = -\frac{\dot{\theta} (1 + e \cos \theta)}{1 + e^2 + 2e \cos \theta} \quad (24)
\]

a further differentiation of which yields the sought inertial pitch acceleration command \(\ddot{\alpha}_2c\)

\[
\ddot{\alpha}_2c = \frac{\dot{\theta}^2 e \sin \theta (1 + 3e^2 + 4e \cos \theta)}{(1 + e^2 + 2e \cos \theta)^2} \quad (25)
\]

With the pitch command \(\dot{\alpha}_2\) and the inertial pitch-rate command \(\omega_2\), the angle of attack is commanded to be equal to the constant bias angle \(\alpha_{ak,b}\). The actual angle of attack, of course, will depend on the performance of the IPFM reaction jet controller used here. \(^5\)

B. Numerical Results

In the initial phase of aerobraking, the altitude of periapsis \(h_p\), and of apoapsis \(h_a\), are \(h_p = 105\) km and \(h_a = 33,264\) km, whereas in the final phase the apoapsis altitude decreases to \(h_a = 768\) km. In the initial phase, the atmospheric torque (Fig. 6) caused by the roll and yaw drag forces is significant for nearly 10 ms centered around the periapsis. In the final phase, this duration expands to 20 min.

while the filtered rate \(\omega_{2f}\) nears zero, \(\omega_2\) oscillates with a small amplitude (see Fig. 3) over the bias rate \(\omega_{2b} = 3.24\) deg/s.
becomes approximately equal to \( \theta_{d}\), and that the pitch rate \( \dot{\phi}_z \) tracks \( \dot{\xi} \). In view of the constraints of given thruster parameters, the performance of the controller is thus very satisfactory. Reference 3 may be reviewed for further details.

V. Soft Martian Landing Controller

When a spin-stabilized interplanetary spacecraft approaches Mars, it is despun and, as Martian atmosphere becomes palpable, the tumbled parachute is deployed, reducing the approach velocity to a constant terminal velocity, say 80 m/s, with possibly a sizable local horizontal component. The parachute is jettisoned at an altitude as low as the thrusters onboard and landing dynamics would permit, and both horizontal and vertical velocity components are reduced to almost zero by an active landing controller in the remaining altitude before touchdown. The controllers developed here accomplish this purpose and are significantly enhanced versions of the classical longitudinal and lateral controllers\(^{9,10}\) for lunar landing.

A. Longitudinal Controller

The objective of this controller is to reduce the vertical velocity at the time of parachute jettisoning (at, Fig. 8) to a specified, low, constant velocity \( v_h \) (1 m/s) by a certain altitude \( h_c \) (5 m, point b, Fig. 8) above the destination planet surface. An additional objective of the controller is to lower the spacecraft at a constant velocity from the point b to touchdown. In Refs. 9 and 10, the controller is turned off at a slight altitude (point c, Fig. 8), e.g., 1 m, to avoid raising dust with thruster plume and thereby contaminating the planetary surface. However, this requires an accurate altimeter to detect the point c, escalating the spacecraft cost and weight. The alternative adopted here is to let the controller bring the spacecraft down at a constant velocity until touchdown, whereupon the inertial reference unit (IRU) indicates zero vertical velocity and the controller then will turn itself off. The nominal free-fall phase c'd hence do not exist in the present controller.

Figure 9 (adapted from Fig. 13.12, p. 583, Ref. 10) depicts the geometry of descent. Let \( a_N \) be the nominal spacecraft deceleration in the body-fixed \(-z_b\) direction produced by selected thrusters. Also, denote the Mars gravitational acceleration along the vertical z axis as \( g_M \). Because the pitch angle \( \alpha \) is small, \(|\dot{\alpha}| < 1\) rad, the net deceleration of the spacecraft along the z axis is essentially \( (a_N - g_M) \). Because the spacecraft's reference trajectory passes through the point \( b(h_c, v_h) \) in the \( h - v_z \) plane, Fig. 8, the commanded vertical velocity \( v_z \) at an altitude \( h \) must be

\[
\dot{z}_c = \dot{v}_z = 2(a_N - g_M)(h - h_c)
\]

This provides a descent contour shown in the control block diagram, Fig. 10. The actual velocity \( \dot{z}_c \), however, measured by a radar at the instantaneous spacecraft altitude or estimated by an IRU system, is usually different from \( \dot{z}_c \). A noisy velocity error then is obtained as shown and filtered to arrive at a velocity error \( e_N \). To eliminate this error, the reference acceleration \( a_{N} \) is augmented by an incremental acceleration \( K_c e_N \) and the thrusters are turned on appropriately. A new velocity is measured or estimated again and the above procedure repeats itself. To determine a suitable value of the control gain \( K_c \), note that the reference time \( T_0 \) required to bring the initial vertical velocity \( z_0 \) to \( v_N \) is

\[
T_0 = (z_0 - v_N)/(a_N - g_M)
\]

Reference 9 shows that the error \( \delta \) between the actual and the current reference altitude decreases exponentially and 

\[
K_c T_0 > 5 \text{ is a suitable gain because } \exp(-5) = 0.0067.
\]

The nominal deceleration \( a_N \) is determined from Eq. (26), knowing the initial velocity \( z_0 \) and the nominal initial altitude \( h_{\text{nom}} \) at which the landing controller is turned on:

\[
a_N = \frac{\dot{z}_c^2 - \dot{v}_z^2}{2(h_{\text{nom}} - h_c)} + g_M
\]

This is the minimum deceleration that the thrusters must supply. If they deliver more, their width is modulated so as to track \( a_N \) with little error.

The above controller is used also for the constant-velocity phase bc (Fig. 8) executed near ground. This phase is introduced because, otherwise, the lateral controller, discussed below, causes large pitch rate and pitch acceleration commands near the landing site. To effect a constant-velocity descent, the nominal acceleration \( a_N \) is set equal to \( g_M \) in the descent contour (26) when the measured altitude \( h \) with measurement errors) is less than or equal to the specified altitude \( h_c \) (e.g., 5 m). Also, the nominal duration \( T_{N} = h_c/v_N \) of the constant-velocity phase is smaller than the time duration \( T_0 \) and therefore the acceleration gain \( K_c \) for this phase is set to be much larger than what \( K_c T_0 = 5 \) would allow.

B. Lateral Controller

The objective of this controller is to reduce to zero the initial horizontal velocity of the spacecraft using the same thrusters that concurrently decelerate the lander in the vertical direction. Consequently, the lander must be pitched for lateral control by an angle \( \phi \) (Fig. 9), not necessarily the same as the angle \( \beta \) the angle of the velocity vector with the vertical. When the pitch angle \( \theta \) is equal to \( \beta \), that results in gravity turn—a lateral control scheme with certain advantages elucidated and utilized subsequently. The block diagram of the lateral controller is shown in Fig. 11. It is an amended version of the controller in Fig. 13.13 of Ref. 10 wherein the commanded pitch rate \( \dot{\phi} \) is proportional to \( v_h/R \), where \( v_h \) is the lateral velocity and \( R \) is the slant range (Fig. 9). Though \( \dot{\phi} \) is calculated is effective initially, the scheme slowly degrades inasmuch as it causes the spacecraft to swing while approaching the ground, with \( \dot{\phi} \) experiencing divergent oscillations. Because this is unsatisfactory, \( \dot{\phi} \) is calculated instead from a suitable altitude downward (40 m presently) according to gravity turn, as portrayed in Fig. 11. The advantage of doing so is that the divergent oscillations in the pitch-rate command then are replaced by stable, smooth commands. The pitch command for gravity turn, in the linear regime, is \( \dot{\phi} = \dot{\phi}_c \approx \dot{\phi}_g \). But even the gravity turn has its limitations because, as the spacecraft reaches very near the ground, both lateral and longitudinal velocity components approach zero and the rate and acceleration commands \( \dot{\phi}_c \) and \( \ddot{\phi}_c \) become intolerably large. The lateral controller therefore is turned off altogether when \(|x| < 0.1 \text{ m/s} \), accepting the residual small pitch angle and rate, and residual lateral velocity (1–1.5 m/s) and longitudinal velocity (0.5–2 m/s) to be withstood by the spacecraft structure and the resident electronic components.
Also, these touchdown conditions and the center of gravity of the lander are stipulated to be such that the lander does not overturn.

Regarding the gain $K_{\text{lat}}$ in Fig. 11, the pitch rate command $\theta_{\text{yc}}$ in linear regime (that is, small horizontal velocity compared to the longitudinal velocity and small pitch angle), according to Pfeffer, is $\theta_{\text{yc}} = K_{\text{lat}}\eta_h/R$, where the angle $\eta$ and the longitudinal velocity $v_l$ are defined in Fig. 9. The gain $K_{\text{lat}}$ is optimal when it renders the two roots of the quadratic equation (13.52) of Ref. 10 equal, leading to

$$K_{\text{lat}} = \frac{b_n - 1}{2} \pm \frac{b_n^2 - 2b_n}{2}$$

(28)

where the positive constant $b_n$ is defined as $b_n = 2a_N/(a_N - g_M) > 2$.

Because the plus sign yields larger $K_{\text{lat}}$ than the minus sign does, we adopt the plus sign and the larger gain.

In a gravity turn, the angle $\eta$ (Fig. 9) is zero by definition and the commanded pitch angle $\theta_{\text{yc}}$ varies continuously as the horizontal and vertical velocity components $x$ and $z$ decrease and the spacecraft descends to the Martian surface. Knowing $x$ and $z$ with the aid of a radar or IRU, the ideal angle $\theta_{\text{yc}}$, rate $\dot{\theta}_{\text{yc}}$, and acceleration $\ddot{\theta}_{\text{yc}}$ can be calculated to control the pitch attitude. This scheme therefore is used below a certain altitude, e.g., 40 m (Fig. 11), replacing the earlier problematic pitch-rate command.

Defining a quantity $\alpha_j = b_n/2 (J \text{ for jets})$, the linear pitch command is found to be

$$\theta_{\text{yc}} = \frac{x_i z^{j-1}}{z^j} = \frac{x}{z}$$

(29)

Comparing $I_2\theta_{\text{yc}} (I_2 = \text{pitch moment of inertia})$ with the available pitch-control torque, the gravity turn can be terminated when the former exceeds the latter, and the constant velocity descent then begins.
C. Numerical Results

The performances of increasingly more complete landing controllers were examined and extensive simulation studies were conducted but, to conserve space, only limited results are shown here. Reference 3 illustrates an unsatisfactory, oscillatory pitch-control performance of the original landing controller of Refs. 9 and 10 without gravity turn for pitch control at low altitudes. Figure 12, in contrast, illustrates performance of the pitch controller when the gravity turn is incorporated below 45-m altitude, provided the horizontal velocity magnitude \( |v_x| \) is greater than 0.1 m/s. The unacceptable divergent oscillations are absent in Fig. 12 because 1) when \( h < 45 \text{ m and } |v_x| > 0.1 \text{ m/s} \), the smooth pitch and pitch rate commands are obtained from Eqs. (29) and (30); 2) when the horizontal velocity magnitude \( |v_x| \leq 0.1 \text{ m/s} \), regardless of the altitude, the commanded pitch angle and rate \( \delta_p, \dot{\delta}_p \) are both zeroed; and 3) the pitch angle and rate commands are both zeroed in the constant-velocity phase below 5 m.

The performance of the landing controller which, in addition to gravity turn, uses a radar altimeter with certain measurement noise characteristics and digital fading memory filters to smooth noisy altitude and velocity measurements is illustrated in Figs. 13 and 14. The initial altitude in these figures is set at 348.84 m, based on a reference constant-velocity duration, \( t_{cv} \), yielding an estimate of the actual touchdown horizontal velocity \( v_h \) is negative, \(-1.32 \text{ m/s}\) (Fig. 13). Because the IRU navigation system operates essentially noise-free below \( h = 45 \text{ m} \), the velocities are seen to be such after \( t = 8 \text{ s} \), where the lander crosses 45-m altitude. The \( h - v_x \) guidance trajectory and actual trajectory are portrayed in Fig. 14, where we observe that the longitudinal controller tracks the guidance trajectory easily during both the constant-velocity phase and the constant-velocity phase, the latter starting at a true altitude of \( 5 + 2.4 \text{ m} \) instead of 5 m due to \(-2.14 \text{ m} \) of \( h_{bias} \) error. Figure 28 in Ref. 3 shows picturesquely the poignant descent of the lander in the altitude-downrange plane. Therein enthralling is its slight horizontal backward motion just before touchdown, preening with pride.

VI. Concluding Remarks

Four attitude controllers are presented for spinning and nonspinning phases of interplanetary spacecraft. For Rhumb-line precession of the spin axis, it is crucial to maintain a constant phase of thruster pulses with the sun. To meet this condition, these pulses must be slightly aperiodic; exactly periodic pulses at spin period would veer the spin axis to an unintended direction. Thrusters can damp very effectively nutations of spacecraft with products of inertia if its transverse angular rates are passed through suitably designed band-pass filters that eliminate constant rate biases. When the spacecraft arrives at the destination planet, the initial highly eccentric orbit
is circularized with aerobraking. Attitude of the spacecraft can be controlled during aerobraking, amid an order-of-magnitude uncertainty in the air density, with IPFM reaction jet controller, provided the reference pitch and pitch-rate commands are inputted to the controller. Finally, the vertical and horizontal velocity controllers available in the literature for soft landing are inadequate because the latter controller causes divergent oscillations in pitch and pitch-rate commands as the spacecraft approaches the ground. This shortcoming is removed by switching at a suitable intermediate altitude, e.g., 50 m, to pitch and pitch-rate commands associated with gravity turn. When very close to the ground, e.g., 5-m altitude, a constant-velocity reference trajectory is introduced in the vertical velocity controller for a soft touchdown of the spacecraft.

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