Obtaining engineering estimates of aerodynamic forces on air-breathing slender bodies

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Abstract

This paper demonstrates simple techniques to obtain quick engineering estimates for the the normal load and moments on slender air-breathing shapes at small angles of attack. Rapid estimates of the normal load can be obtained using a two-dimensional, subsonic, Trefftz plane analysis. The results agree fairly well with experimental data (from NASA and AGARD) for three-dimensional slender bodies in supersonic flows. However, the method is insensitive to variations in the three-dimensional shape of the body and incapable of computing the moments on the body. The pitching moments are therefore computed using a subsonic 3D vortex panel method. These results are also in reasonably good agreement with experimental data.

Keywords: Normal load, pitching moment, air-breathing slender bodies, Trefftz plane analysis, 3D vortex panel method

1 Introduction

There is often a need to obtain quick engineering estimates for air-breathing slender bodies in both subsonic and supersonic flow. The present work demonstrates simple techniques using which rapid estimates of the normal load and pitching moment can be obtained. A subsonic Trefftz plane analysis can be used to compute the normal load. This is a two dimensional analysis and considerably simplifies the problem. The technique is also very efficient. It is however incapable of predicting the moments on the body. In order to compute this a 3D vortex panel method is used. The results obtained by the computations are compared with experimental data from AGARD [1] and NASA [3, 4, 5]. The AGARD and NASA reports provide experimental data for air-breathing slender shapes in supersonic flows.

The Trefftz plane analysis while not presented in its entirety here is an interesting technique that can be used to obtain rapid estimates of the normal load. The simplicity of the method is due to the fact that it requires just a cross-section of the body shape to compute the load. This reduces both the geometric complexity and the computational difficulties associated with a full three-dimensional analysis. The method is capable of handling air-breathing configurations. This analysis is entirely subsonic. The justification for this is that the normal load is dependent on the vorticity distribution in the Trefftz plane. The distribution and shape of the vorticity in the Trefftz plane for highly streamlined bodies is dependent on the cross flow and not on the axial flow. Hence, for Mach numbers less than about 4, at small angles of attack, the cross flow continues to remain subsonic.

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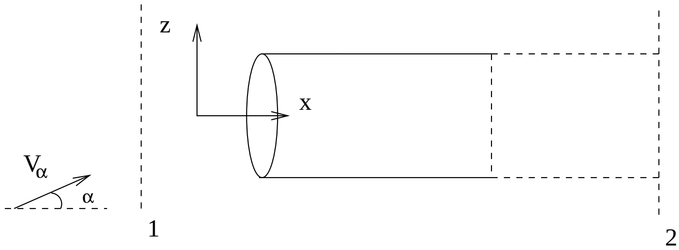

Figure 1: Lifting duct and its trailing vortex system.

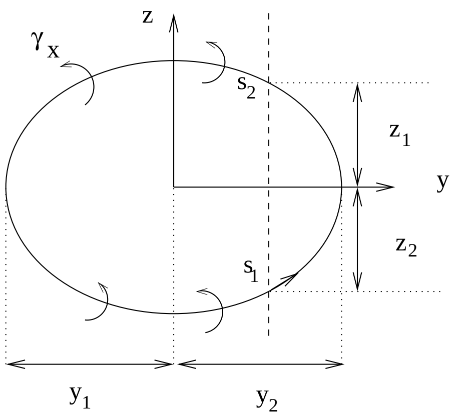

Figure 2: Illustration of quantities used to compute the normal load.

Since the flow remains subsonic this analysis is bound to give reasonable results. This is borne out from the results presented below.

The subsonic 3D panel method is a well researched and fairly old technique but in the present work it is applied to fairly complex air-breathing geometries at supersonic Mach numbers.

2 Methodology

Two methods are used in the present work. A Trefftz plane analysis can be used to obtain the normal load on the body. This technique is subsonic and two-dimensional. The other method used is a 3D vortex panel method to compute the load and pitching moment on the body. This section provides some details on the methodologies used.

2.1 Trefftz plane analysis

The Trefftz plane is an idealized plane far downstream of the body and perpendicular to the axes of the body. This is illustrated for the case of a circular duct in Fig. 1. The plane 2 is the Trefftz plane. It is possible to study the trailing vorticity distribution in the Trefftz plane and obtain the load on a body. Rajan and Shashidhar [10] derived an exact leading term solution for low aspect ratio wings using such a Trefftz plane analysis. Raghavendra [9] used the same idea to obtain the load on a circular duct and a duct with fins. Govindarajan [2] used the approach to compute the normal and suction forces on an air-breathing slender body.

The basic idea behind this technique is that the vorticity distribution in the Trefftz plane for slender bodies depends only on the cross flow. The cross flow then determines the load on the body.

For supersonic flow at small angles of attack, the cross flow Mach number is subsonic. Due to this, it is possible to use a two dimensional Trefftz plane analysis and compute the normal load. Using the Trefftz plane analysis therefore reduces the problem from a three-dimensional one to a two-dimensional one and thereby considerably reduces the computational effort. Hence, given a body, an axial location where the cross sectional area enclosed becomes maximum is chosen. Alternatively, the axial view of the body is projected onto the Trefftz plane and this projected two-dimensional surface is considered. The resulting two-dimensional shape is exposed to the cross flow velocity of $V_\infty \sin\alpha$. The resulting intensity of surface vorticity on the chosen cross section is computed using a vortex panel method. If the axial velocity of the fluid inside the body, is also known, then the normal force can be computed using the Trefftz plane analysis using the following equation,

$$F_z = \rho V_\infty \cos\alpha \int_{y_1}^{y_2} dy \int_{s_1}^{s_2} \gamma_x ds - \rho V_\infty \sin\alpha \int_{y_1}^{y_2} dy \int_{z_1}^{z_2} u_2 dz. \quad (1)$$

The various quantities used in the above are illustrated in Fig. 2. γ_x is the vorticity distribution at the Trefftz plane. The value of u_2 is the perturbation velocity component along the x-axis at the plane 2 and inside the body. The integral that contains u_2 is the one that accounts for the air-breathing nature of the body. The derivation of this expression and other details are given in [11].

In the present work a linear vortex panel method with a no-penetration condition is used to obtain the vorticity distribution for arbitrary cross sections.

The Trefftz plane analysis will clearly be insensitive to changes in the geometry so long as the maximum cross sectional shape remains unchanged. It is also incapable of computing the moments on the body. In order to compute this a 3D vortex panel method is used.

2.2 3D vortex panel method

Katz and Plotkin [6] describe two and three dimensional panel methods in considerable detail. In the present work the body is split into quadrilateral and triangular panels with vortex filaments located along the edges of the panels. The no-penetration condition is applied at the centroid of each panel. The Kutta condition is satisfied by shedding horse-shoe vortices from the trailing edge of the body. The influence matrix is solved using a Singular Value Decomposition (SVD) [8]. The SVD is used instead of the simpler and faster LU decomposition because the SVD produces more reliable results. The shapes considered in the present work have very low aspect ratios. The low aspect ratio along with the trailing edge horse-shoe vorticity tends to make the matrices involved singular. The SVD handles such singular matrices and produces good results. Once the vorticity distribution on the surface of the body is known, the force on each panel is computed using the approach detailed in Konstadinopoulos [7]. This method distributes the concentrated vorticity on each panel and then computes the pressure jump across the panel. Using this, the force distribution on the body is obtained which is then integrated to produce both the total force and the moments on the body. Hence, using the 3D vortex panel method the load and moments on the body can be obtained. This approach is much more complicated than the two-dimensional analysis because of the geometric complexity and the large number of panels required to obtain good solutions.

3 Results and discussion

A few air-breathing configurations are considered and the computed results are compared with experimental data. As an example we also present the results for one of the cases using the Trefftz plane analysis. For the other cases the results of the 3D vortex panel method are provided.

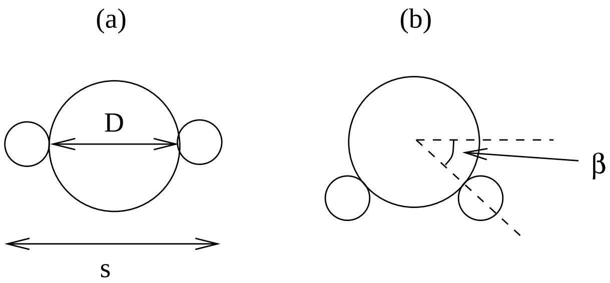

Figure 3: The various parameters used in the computation of $C_{N\alpha}$ for the missile shapes presented in the AGARD report[1].

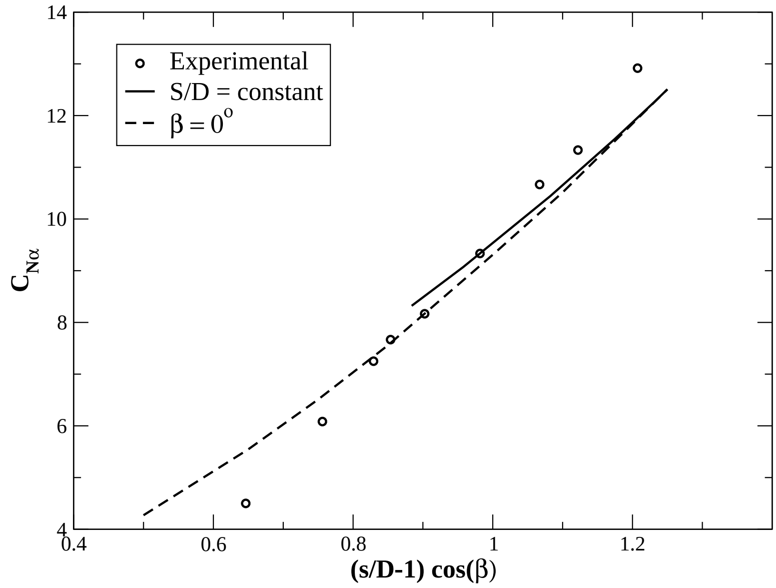

Figure 4: Computed and experimental $C_{N\alpha}$ for the AGARD configurations. The straight line plots the case when $s/D = 2.25$ and β is varied. The dashed line plots the case where $\beta = 0$ and s/D is varied. The experimental value of $M_\infty = 3.2$.

3.1 Comparison with AGARD data

The AGARD report[1] provides data for slender missile like shapes consisting of a solid central body with symmetrically placed axi-symmetric ducts. The cross section of the configuration is shown in Fig. 3. Experimental values of $C_{N\alpha}$ are provided. The Trefftz plane analysis is used to compute the load and is compared with the available experimental data. The results are plotted in Fig. 4. In one case β is held fixed at $0°$ and s/D is varied. In the other case β is varied between 0 and 45 degrees and s/D is held fixed. The experimental data is for the Mach number of 3.2. Fairly good agreement is seen.

For the case where $\beta = 10°$, $s/D = 2.25$, the number of panels is doubled and the convergence of the results is studied. The Grid Convergence Index (GCI)[12] for the coarse grid is computed as,

$$GCI = \frac{3r^p}{r^p - 1}\frac{s_1 - s_2}{s_1}, \tag{2}$$

where s_1 is the solution at the finer grid, s_2 at the coarser grid, r is the ratio of the grid sizes, h_2/h_1 and p is the order of the method. In the present case, since the number of panels is doubled, $r = 2$,

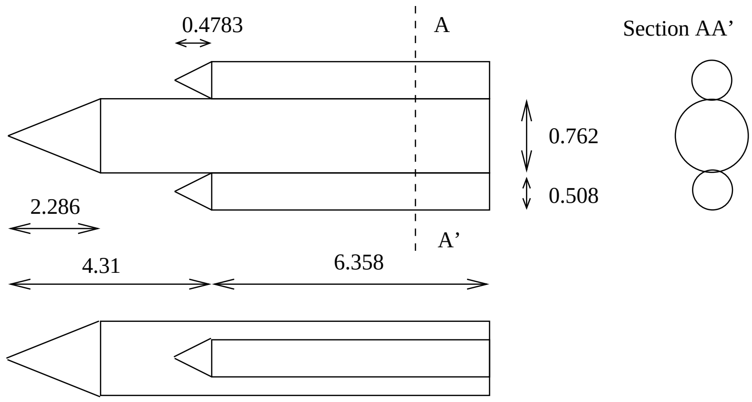

Figure 5: Cross section of the NASA B1I1 [3] configuration (all dimensions in centimeters).

and p is assumed to be 2. Given this we find that the $GCI = 0.0071$, which indicates very good convergence.

3.2 Comparison with NASA data

The NASA experiments of Clyde Hayes [3, 4, 5] provide data on slender air-breathing missile configurations. Several different configurations are considered for subsonic and supersonic speeds. The configurations include twin axi-symmetric and two-dimensional air-intakes. Experimental data for the normal load and pitching moment are available in the range $2.5 \leq M_\infty \leq 3.95$. A few of these are considered for comparison with the computations. The actual geometries are complex and for the computations shown here, simplified versions of the actual geometries are considered.

The B1I1 configuration as per the NASA report [3] consists of two axi-symmetric air-intakes. The geometry considered for computation is shown in Fig. 5. The results of the computations using the three-dimensional panel method for C_N and C_M are plotted in Figs. 6 and 7 respectively. The results are plotted for the case where the intakes are treated as fully closed and fully open. This is done because data on the mass flow rate in the experiments was unavailable. The results are in good agreement. It is also seen that for low angles of attack the experimental results are not strongly dependent on the Mach number.

As the next case, the NASA B1I2 configuration is considered. This configuration has twin two-dimensional intakes on the sides. The configuration chosen for computation is shown in Fig. 8. The results of the 3D panel method applied to the B1I2 configuration for C_N and C_M are shown in Figs. 9 and 10 respectively. In this case there is a shift in the values for the 3D panel method results as compared to the experimental data. This is because of the way in which the geometry is approximated. However, it is seen that the computed results show the right trends at small angles of attack. It is also seen that the experimental results deviate from the computations at smaller angles of attack than in the B1I1 case. This is likely to be due to the separation of the flow at the sharp edges of the sides of the air-intakes. This separation and possible vortex formation is bound to increase the lift non-linearly. The 3D vortex panel method used for the present results do not take separation into account and therefore do not capture the non-linear load.

For the B1I1 and B1I2 cases, the 3D panel method computations used a relatively small number of panels (of the order of 750 - 1000) in order to keep the computational time small. When the number of panels for the B1I2 case was quadrupled (doubled along each direction) it was found that the normal load values dropped by about 12% (producing a coarse grid GCI of about 0.495). However, the computational time increased by a large amount because the SVD computation takes a long while, and a large amount of memory. The fact that a 12% change is seen in the normal

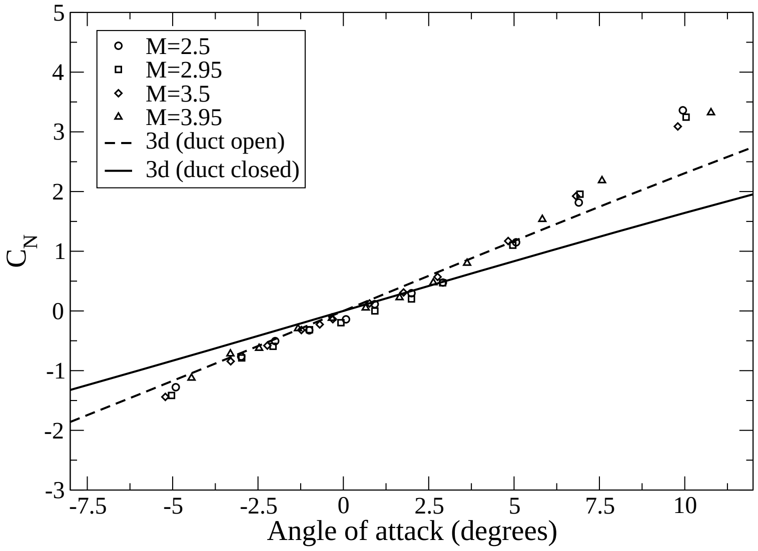

Figure 6: C_N versus angle of attack for the the NASA B1I1 configuration with the ducts placed on the side of the body as shown in Fig.5. The results are computed using the 3d vortex panel method.

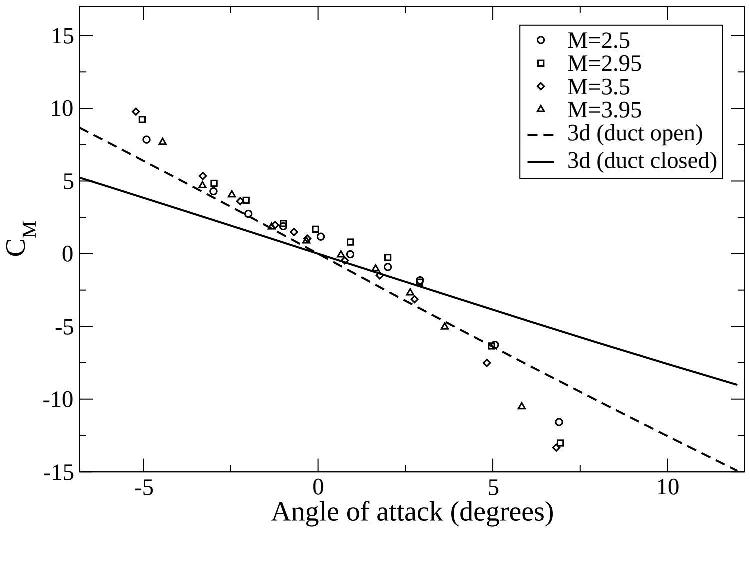

Figure 7: C_M versus angle of attack for the the NASA B1I1 configuration with the ducts placed on the side of the body as shown in Fig.5. The results are computed using the 3d vortex panel method.

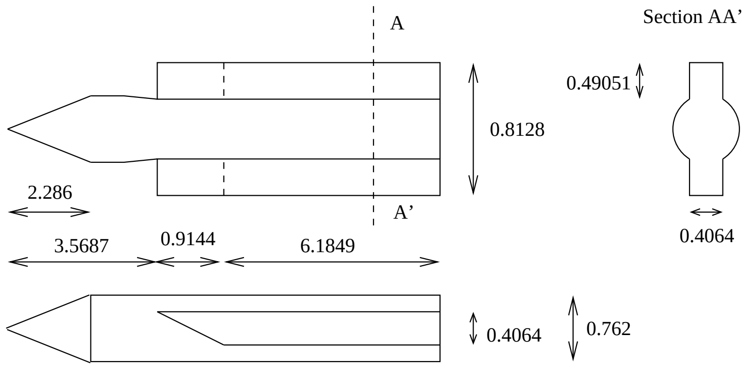

Figure 8: Simplified NASA B1I2 geometry considered for the computation. All dimensions in centimeters.

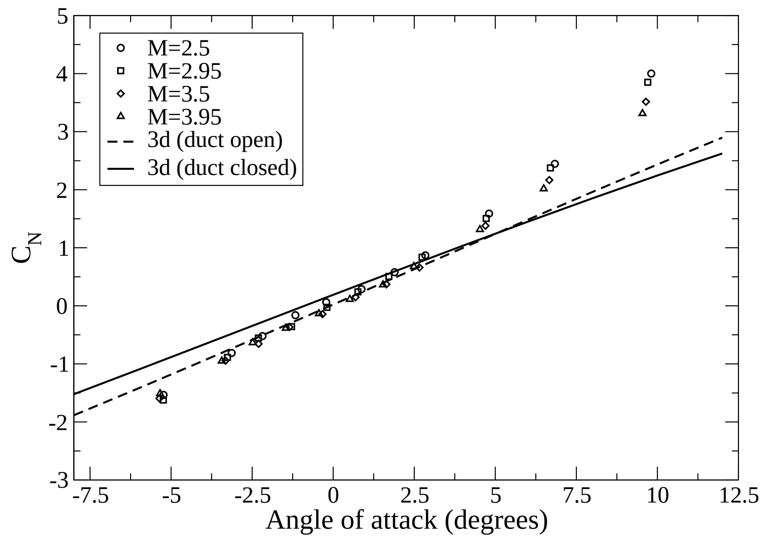

Figure 9: C_N versus angle of attack for the the NASA B1I2 configuration with the ducts placed on the side of the body as shown in Fig.8. The results are computed using the 3d vortex panel method.

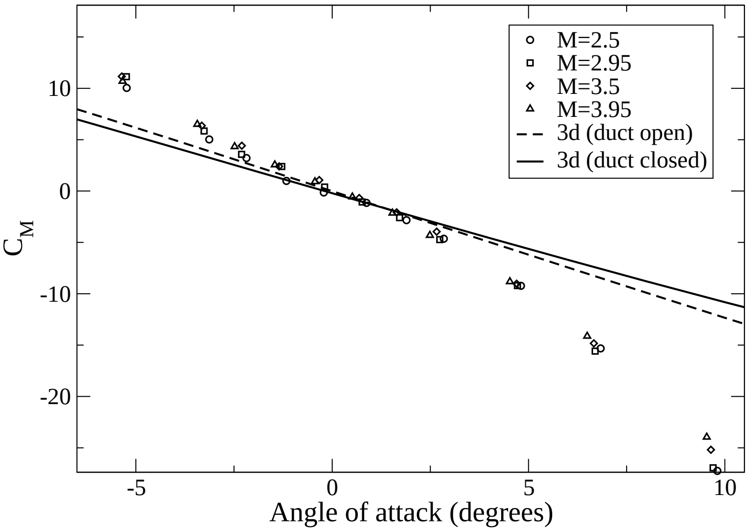

Figure 10: C_M versus angle of attack for the the NASA B1I2 configuration. The results are computed using the 3d vortex panel method.

load indicates only reasonable convergence. The computations are reasonably close to the 2D results presented in [11]. Performing computations using such large number of panels would require much larger computational resources. Hence, the 3D panel method results are to be taken as indicative of the trends and not as definitive results. These trends appear correct and agree well with the experimental data.

It must be mentioned that the 2D computations (Trefftz plane analysis) typically take less than a few seconds to complete for a given configuration on a Pentium 4 machine running at 1.7 GHz. The 3D panel method however takes significantly more time (of the order of several minutes) due to the large number of panels and the SVD computation.

4 Conclusions

In this paper a simple technique that can be used to compute the normal load on an air-breathing slender body has been demonstrated based on the work of [11]. This technique is simple but is incapable of computing the pitching moment on the body. In order to compute this a 3D vortex panel method is used. The computed results are compared with experimental data and the agreement is found to be good. The general trends are captured and the values are reasonably close to available experimental data. It is noticed that the computations are successful in predicting the load and moment even for supersonic flows, though the computational method is purely subsonic. Since 3D panel methods are well established, these results indicate that it is certainly possible to use simple and well studied techniques to compute engineering estimates for air-breathing, low aspect ratio configurations at small angles of attack exposed to supersonic mean flow.

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